B.Sc. DEGREE EXAMINATION,NOVEMBER 2019
I Year II Semester
Allied Mathematics - II

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define countable and uncountable set.
2. Prove that the set of all integers is countable.
3. Define the convergence of a sequence.
4. If $\sum_{n=1}^{\infty} a_n$is a convergent series, prove that $\lim_{n\to\infty} a_n = 0$.
5. State Rolle's theorem.
6. Find the Taylor series about $x = 2$ for $f(x) = x^3 + 2x + 1$, $x \in (-\infty, \infty)$.
7. Prove that $L\left[e^{-at}\right] = \frac{1}{s+a}, s + a > 0$.
8. Find $L\left[\cos^2 6t\right]$.
9. Find $L^{-1}\left[\frac{6}{(s+2)^4}\right]$.
10. Find $L^{-1}\left[\frac{s}{(s-2)^2+4^2}\right]$.
11. Evaluate $\lim_{n\to\infty} \frac{3n^2-6n}{5n^2+4}$.
12. Show that $L^{-1}\left[\frac{s+1}{s^2+2s+2}\right] = e^{-t}\cos t$.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove that the interval $[0, 1]$is uncountable.
14. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$is divergent.
15. If $f'(x) = 0$ for all $x \in [a, b]$, prove that f is constant on $[a, b]$.
16. Find $L\left[e^{7t}\sin^2 t\right]$.

17. Find $L^{-1}\left[\dfrac{3s+5}{s(s-2)(s+3)}\right]$.
18. If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges, prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.
19. Find $L[\cos 4t\ \sin 2t]$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If $A_1, A_2, \cdots \cdot A_n$ are countable sets, then prove that $\bigcup\limits_{n=1}^{\infty} A_n$ is countable.
21. Prove that a non decreasing sequence which is bounded above is convergent.
22. If f is a real valued function on $[a, a+h]$ such that $f^{n+1}(x)$ exists for every $x \in [a, a+h]$ and f^{n+1} is continuous on $[a, a+h]$ then prove that
$$f(x) = f(a) + \frac{f'(a)}{\angle 1}(x-a) + \frac{f''(a)}{\angle 2}(x-a)^2 + \cdots \cdot + \frac{f^{(n)}(a)}{\angle n}(x-a)^n + R_{n+1}(x),$$
where $R_{n+1}(x) = \dfrac{1}{\angle n}\displaystyle\int_a^x (x-t)^n f^{(n+1)}(t)\,dt$.
23. Find (i) $L[\cos at]$ (ii) $L[\sin^3 2t]$.
24. Find $L^{-1}\left[\dfrac{s^2+9s+2}{(s-1)^2(s+3)}\right]$.

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