M.Sc. DEGREE EXAMINATION,NOVEMBER 2019
I Year I Semester
Algebra I

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. State the second sylow theorem.
2. Define Normalizer of a group.
3. Define solvable group.
4. Define cyclic module.
5. Define trace of T.
6. Define Hermitian adjoint of T.
7. Give an example for a finite field.
8. Define finitely generated module.
9. If the field F has p^m elements prove that F is the splitting field of the polynomial x^{p^m}- x .
10. Let Q be the division ring of real Quaternions. Find the adjoint of x = α_0+α_1i+α_2j+α_3k in Q.
11. State Left Division Algorithm.
12. If T is unitary,then prove that $TT^* = 1$.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Show that a group of order p^2, where p is a prime number , is abelian .
14. Let G be a group and suppose that G is the internal direct product of N_1 ....N_r . Let T = $N_1 \times \cdots \times N_r$. Prove that G and T are isomorphic.
15. If N is normal and if $vN^* = 0$, then prove that v N = 0.
16. Prove that for every prime number p and every positive integer m there exists a field having p^m elements.
17. If a $\in$ H then prove that $a^{-1} \in$ H if and only if N(a) = 1
18. State and prove Lagrange identity.
19. Prove that G is solvable if and only if $G^{(k)}$ = (e) for some integer k .

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Derive the class equation of a finite group.
21. State and prove the fundamental theorem on finitely generated modules over Euclidian rings.
22. Prove that the linear transformation T on V is unitary if and only if takes orthonormal basis of V into an orthonormal basis of V.
23. State and prove Wedderburn theorem.
24. Let C be the field of complex numbers and suppose that the division ring D is algebraic over C. Then prove that D = C.

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