M.Sc. DEGREE EXAMINATION,NOVEMBER 2019
I Year II Semester
Algebra II

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Extension field.
2. What is an algebraic number ?
3. Define splitting field.
4. Define simple extension.
5. Define fixed field.
6. What is Galois group ?
7. Define index of nilpotence.
8. When do we say that the two linear Transformations are similar ?
9. Define Jordan block.
10. Define characteristic polynomial.
11. Define normal extension.
12. Define rational canonical form.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If L is an algebraic extension of K and if K is an algebraic extension of F then prove that L is an algebraic extension of F.
14. If $f(x) \in F[x]$ be of degree $n \geq 1$. Then prove that there is an extension E of F of degree at most n! in which *f(x)* has n roots.
15. Show that the fixed field of G is a subfield of K.
16. If $T \in A(V)$ is nilpotent , then prove that $\alpha_0 + \alpha_1 T + \ldots\ldots\ldots\ldots\ldots\ldots\ldots +$ $\alpha_n T^m$where the $\alpha_i \in F$ is invertible if $\alpha_0 \neq 0$.
17. If T in $A_F(V)$ has a minimal polynomial $p(x) = q(x)^e$,where $q(x)$is monic, irreducible polynomial in F[x] ,then prove that a basis of V over F can be found in which the matrix of T of the form

$$\begin{pmatrix} C(q(x)^{e_1}) & & & \\ & C(q(x)^{e_2}) & & \\ & & \ddots & \\ & & & C(q(x)^{e_r}) \end{pmatrix}$$

18. If $f(x) \in F[x]$,then prove that there is a finite extension E of F in which $f(x)$ has a root.
19. If $u \in V_1$ is such that $uT^{n_1-k} = 0$,where $0 < k \leq n_1$,then prove that $u = u_0T^k$ for some $u_0 \in V_1$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that the number 'e' is transcendental.
21. Show that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non trivial common factor.
22. If K is a finite extension of F ,then prove that G (K, F) is a finite group and its order o(G(K, F)) satisfies o(G(K,F)) $\leq$ [k : F].
23. If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.
24. Show that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.

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