M.Sc. DEGREE EXAMINATION,NOVEMBER 2019
II Year III Semester
Differential Equations

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Analytic function.
2. Define Regular singular point.
3. What is fundamental matrix?
4. Solve
   $x'_1 = 5x_1 - 2x_2$
   $x'_2 = 2x_1 + x_2$
5. State Lipschitz condition.
6. State Gronwall inequality to establish the uniqueness of solutions.
7. Eliminate the arbitrary function **f** from the relation $z = xy + f(x^2 + y^2)$
8. What is complete Integral of the first order partial differentia equations.
9. Solve $\left(D^2 - 5DD' + 6D'^2\right) Z = 0$
10. Classify the equation $u_{xx} + u_{yy} = 0$
11. When a partial differential equations is said to be reducible?
12. Find the first two Legendre polynomials.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13 . If $P_m\left(t\right) and\ P_n(t)$ are Legendre Polynomials then prove that
$$\int_{-1}^{1} P_n\left(t\right) P_m\left(t\right) dt = 0\ if\ \ m \neq n$$

14. Find the fundamental matrix for the system $x' = Ax$ where $A = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix}$
    where $\alpha_1, \alpha_2, \alpha_3$ are constants.
15. Using the method of successive approximation find the solution of the initial value problem $x' = -x, x\left(0\right) = 1, t \geq 0$

16. Show that the equations $\mathbf{x}p - yq = x$, $x^2p + q = xz$ are compatible and find their solution.
17. Solve the equations $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$
18. Solve the equation $\left(2D^2 - 5DD' + 2D'^2\right) z = 5\ sin(2x + y)$
19. Let $\Phi(t)$,$t \in I$, define a fundamental matrix of the system $x' = Ax$ such that $\Phi\,(0) = \mathrm{E}$, where A is a constant matrix and E denotes the identity matrix. Then prove that Φ satisfies $\Phi\,(\mathrm{t} + \mathrm{s}) = \Phi\,(\mathrm{t})\,.\Phi(\mathrm{s})$ for all values of t and s∈I.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Find the power series solution of Bessel's equation $t^2x'' + tx' + \left(t^2 - p^2\right) x = 0$ of order p, where p is a constant.
21. State and prove Existence and Uniqueness theorem.
22. State and prove Picard's theorem.
23. Using Charpit's method to solve $p = (z + qy)^2$
24. Reduce the equation $(n-1)^2\frac{\partial^2 z}{\partial x^2} - y^{2n}\frac{\partial^2 z}{\partial y^2} = ny^{2n-1}\frac{\partial z}{\partial y}$ to canonical form and find its general solution.

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