

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019
II Year IV Semester
Differential Geometry and Tensor Calculus

Time : 3 Hours

Max.marks :75

Section A ($10 \times 2 = 20$) Marks

Answer any **TEN** questions

1. Define curvature.
2. Define osculating circle.
3. What is an helicoid?
4. Calculate the first fundamental coefficient E,F,G and H for the surface $r = (u, v, u^2 - v^2)$.
5. Define convex region.
6. Write the expression for K_g .
7. Define covariant and contravariant tensor of rank one.
8. What are symmetric and skew symmetric tensors.
9. Define associated tensors.
10. Define christoffels symbols of first and second kind.
11. What is the condition for orthogonality of the two families of curves.
12. State Gauss Bonnet Theorem.

Section B ($5 \times 5 = 25$) Marks

Answer any **FIVE** questions

13. Derive the equation of osculating plane at a point P.
14. For the anchor ring $r = ((b+acosu)\cos v, (b+acosu)\sin v, a\sin u)$, calculate the area corresponding to the domain $0 \leq u \leq 2\pi, 0 \leq v \leq 2\pi$.
15. Prove that the characteristic property of a geodesic is that at every point its Principal normal is normal to the surface
16. Prove that if all the components of a tensor vanish in one coordinate system , then they necessarily vanish in all other admissible coordinate systems.
17. State and prove Ricci's theorem.
18. Define involute and derive the involute of a space curves.
19. A helicoids is generated by a screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoid.

Section C ($3 \times 10 = 30$) MarksAnswer any **THREE** questions

20. Obtain the curvature and torsion of the curve of intersection of the two quadric surfaces $ax^2 + by^2 + cz^2 = 1$, $a'x^2 + b'y^2 + c'z^2 = 1$
21. On the paraboloid $x^2 - y^2 = z$, Find the orthogonal trajectories of the sections by the planes $z = \text{constant}$.
22. Define geodesic and derive the geodesic differential equation.
23. State and prove quotient laws.
24. Prove that the christoffel's symbols of the second kind are not tensors unless the coordinate transformations is affine.

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019
II Year IV Semester
Differential Geometry and Tensor Calculus

Time : 3 Hours

Max.marks :75

Section A ($10 \times 2 = 20$) Marks

Answer any **TEN** questions

1. Define curvature.
2. Define osculating circle.
3. What is an helicoid?
4. Calculate the first fundamental coefficient E, F, G and H for the surface $r = (u, v, u^2 - v^2)$.
5. Define convex region.
6. Write the expression for K_g .
7. Define covariant and contravariant tensor of rank one.
8. What are symmetric and skew symmetric tensors.
9. Define associated tensors.
10. Define christoffels symbols of first and second kind.
11. What is the condition for orthogonality of the two families of curves.
12. State Gauss Bonnet Theorem.

Section B ($5 \times 5 = 25$) Marks

Answer any **FIVE** questions

13. Derive the equation of osculating plane at a point P.
14. For the anchor ring $r = ((b + a \cos u) \cos v, (b + a \cos u) \sin v, a \sin u)$, calculate the area corresponding to the domain $0 \leq u \leq 2\pi, 0 \leq v \leq 2\pi$.
15. Prove that the characteristic property of a geodesic is that at every point its Principal normal is normal to the surface
16. Prove that if all the components of a tensor vanish in one coordinate system, then they necessarily vanish in all other admissible coordinate systems.
17. State and prove Ricci's theorem.
18. Define involute and derive the involute of a space curves.
19. A helicoids is generated by a screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoid.

Section C ($3 \times 10 = 30$) MarksAnswer any **THREE** questions

20. Obtain the curvature and torsion of the curve of intersection of the two quadric surfaces $ax^2 + by^2 + cz^2 = 1$, $a'x^2 + b'y^2 + c'z^2 = 1$
21. On the paraboloid $x^2 - y^2 = z$, Find the orthogonal trajectories of the sections by the planes $z = \text{constant}$.
22. Define geodesic and derive the geodesic differential equation.
23. State and prove quotient laws.
24. Prove that the christoffel's symbols of the second kind are not tensors unless the coordinate transformations is affine.