M.Sc. DEGREE EXAMINATION,NOVEMBER 2019
II Year III Semester
Differential Equations

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define an analytic function.
2. Define regular singular point.
3. What is a fundamental matrix?
4. Define exponential of a matrix.
5. State the Lipschitz condition.
6. State Picard's theorem.
7. Write the Burger equation.
8. Define a complete integral.
9. Define a reducible operator.
10. State the n_{th} th Legendre polynomial.
11. State the condition for a partial differential equation to be elliptic.
12. Define the Gamma function.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove that $\int_{-1}^{1} P_n^2\ (t)\, dt = \ \frac{2}{2n+1}$
14. Find e^{At} when $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.
15. The error $x\,(t) - x_n\ (t)$ satisfies the estimate

    $|x\,(t) - \ x_n(t)|\ \leq \frac{L(Kh)^{n+1}}{K(n+1)!}e^{Kh};\, t \in [t_0, t_0 + h]\,.$
16. Show that the equations $xp - yq = x;\ x^2p + q = xz$ are compatible and find the solution.
17. If $u = f\,(x + iy) + g\,(x - iy)$ where f and g are arbitrary functions, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$

18. Construct an operator adjoint to the Laplace operator given by $L(u) = u_{xx} + u_{yy}$.
19. Use Charpit's method to solve the partial differential equation $\left(p^2 + q^2\right) y = qz$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Solve the Legendre equation $\left(1 - t^2\right) x'' - 2tx' + p\,(p+1)\,x = 0$.
21. Let $A(t)$ be an $n \times n$ matrix that is continuous in t on a closed and bounded interval I. Then there exists a solution to the IVP $x' = A(t)\,x$, $x(t_0) = x_0$; $(t,\ t_0 \in I)$ and this solution is unique.
22. State and prove Picard's theorem.
23. Find the characteristics of the equation $pq = z$, and determine the integral surface which passes through the parabola $x = 0,\ y^2 = z$.
24. Verify the Green's function for the equation $\frac{\partial^2 z}{\partial x \partial y} + \frac{2}{x+y}\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) = 0$ subject to $z = 0; ; \frac{\partial z}{\partial x} = 3x^2$ on $y = x$ is given by $w(x, y, \xi, \eta) = \frac{(x+y)\{2xy + (\xi - \eta)(x-y) + 2\xi\eta\}}{(\xi+\eta)^3}$ and obtain the solution in the form $w = (x-y)\left(2x^2 + xy + 2y^2\right)$

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