

**M.Sc. DEGREE EXAMINATION, NOVEMBER 2019**  
**I Year II Semester**  
**Complex Analysis**

**Time : 3 Hours**

**Max.marks :75**

**Section A** ( $10 \times 2 = 20$ ) Marks

Answer any **TEN** questions

1. Define winding number of a closed rectifiable curve in  $\mathbb{C}$ .
2. Define a simply connected region and give an example.
3. Define essential singularity and give an example.
4. Define residue of an analytic function  $f(z)$  at an isolated singularity  $z = a$ .
5. Define gamma function .
6. Define critical strip and state Riemann hypothesis.
7. Define Poisson kernel.
8. Define Perron family for a continuous function  $f$ .
9. Define order of an entire function  $f$ .
10. Define Landau's constant  $L$ .
11. Define FEP homotopic for two curves in a region  $G$ .
12. Define a Dirichlet region and give an example.

**Section B** ( $5 \times 5 = 25$ ) Marks

Answer any **FIVE** questions

13. State and prove Liouville's theorem.
14. If  $f$  has an essential singularity at  $z=a$ , then show that for every  $\delta > 0, \{f[\text{ann}(a;0,\delta)]\}^{-1} = \mathbb{C}$ .
15. If  $\text{Re} z_n > -1$ , then prove that the series  $\sum \log(1 + z_n)$  converges if and only if the series  $\sum z_n$  converges absolutely.
16. State and prove Harnack's inequality.
17. If  $f$  is analytic in  $D = \{z : |z| < 1\}$ ,  $f(0) = 0$ ,  $f'(0) = 1$  and  $|f(z)| \leq M$  for all  $z$  in  $D$ , then show that  $M \geq 1$  and  $f(D) \supset B(0; \frac{1}{6M})$ .
18. If  $f$  is analytic in the disk  $B(a;R)$  and suppose that  $\gamma$  is a closed rectifiable curve in  $B(a;R)$ , then prove that  $\int_{\gamma} f = 0$ .

19. If  $u : G \rightarrow \mathbb{R}$  is a continuous function which has the MVP, then show that  $u$  is harmonic.

**Section C** ( $3 \times 10 = 30$ ) Marks

Answer any **THREE** questions

20. State and prove the first version of Cauchy's integral formula.
21. State and prove residue theorem.
22. Obtain the factorization of sine function.
23. State and prove the Harnack's theorem.
24. State and prove Schottky's theorem.

**M.Sc. DEGREE EXAMINATION, NOVEMBER 2019**  
**I Year II Semester**  
**Complex Analysis**

**Time : 3 Hours**

**Max.marks :75**

**Section A** ( $10 \times 2 = 20$ ) Marks

Answer any **TEN** questions

1. Define winding number of a closed rectifiable curve in  $\mathbb{C}$ .
2. Define a simply connected region and give an example.
3. Define essential singularity and give an example.
4. Define residue of an analytic function  $f(z)$  at an isolated singularity  $z = a$ .
5. Define gamma function .
6. Define critical strip and state Riemann hypothesis.
7. Define Poisson kernel.
8. Define Perron family for a continuous function  $f$ .
9. Define order of an entire function  $f$ .
10. Define Landau's constant  $L$ .
11. Define FEP homotopic for two curves in a region  $G$ .
12. Define a Dirichlet region and give an example.

**Section B** ( $5 \times 5 = 25$ ) Marks

Answer any **FIVE** questions

13. State and prove Liouville's theorem.
14. If  $f$  has an essential singularity at  $z=a$ , then show that for every  $\delta > 0, \{f[\text{ann}(a;0,\delta)]\}^{-1} = \mathbb{C}$ .
15. If  $\text{Re} z_n > -1$ , then prove that the series  $\sum \log(1 + z_n)$  converges if and only if the series  $\sum z_n$  converges absolutely.
16. State and prove Harnack's inequality.
17. If  $f$  is analytic in  $D = \{z : |z| < 1\}$ ,  $f(0) = 0$ ,  $f'(0) = 1$  and  $|f(z)| \leq M$  for all  $z$  in  $D$ , then show that  $M \geq 1$  and  $f(D) \supset B(0; \frac{1}{6M})$ .
18. If  $f$  is analytic in the disk  $B(a;R)$  and suppose that  $\gamma$  is a closed rectifiable curve in  $B(a;R)$ , then prove that  $\int_{\gamma} f = 0$ .

19. If  $u : G \rightarrow \mathbb{R}$  is a continuous function which has the MVP, then show that  $u$  is harmonic.

**Section C** ( $3 \times 10 = 30$ ) Marks

Answer any **THREE** questions

20. State and prove the first version of Cauchy's integral formula.
21. State and prove residue theorem.
22. Obtain the factorization of sine function.
23. State and prove the Harnack's theorem.
24. State and prove Schottky's theorem.