M.Sc. DEGREE EXAMINATION,NOVEMBER 2019
II Year IV Semester
Functional Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. If N is a normed linear space,then show that norm is a continuous function on N.
2. If N is a normed lineare space and $T, T' \in B(N)$,prove that $\|TT'\| \leq \|T\|\,\|T'\|$.
3. Define a Hilbert space.
4. Define a complete orthonormal set.
5. If A_1 and $A_2$are self adjoint operators on H,Show that their product A_1A_2 is self adjoint if and only if $A_1A_2 = A_2\ A_1$.
6. Define normal operator in a Hibert space.
7. Define topological zeros in a Banach algebra.
8. Define the radical of a Banach algebra A and a semi simple algebra.
9. Define Gelfand mapping.
10. Define Banach * algebra.
11. Define isometric isomorphism.
12. Define the maximal ideal space of a commutative Banach algebra.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Let M be a closed linear subspace of a normed linear space N.If the norm of a coset x + M of N/M is defined by $\|x + M\|$=inf $\{\|x + m\|$: m $\in$ M$\}$,then show that N/M is a normed linear space.
14. If M is a closed linear subspace of a Hilbert space H, then prove that H $= M \oplus M^{\perp}$.
15. If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other,then show that $N_1 + N_2$ and $N_1\ N_2$ are normal.
16. If r is an element of R,then prove that 1−r is regular.
17. Show that the maximal ideal space m is a compact Hausdorff space.

18. State and prove Minkowski's inequality.
19. If x is a normal element in a B* algebra, show that $\left\|x^2\right\| = \|x\|^2$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. State and prove Hahn- Banach theorem.
21. State and prove Uniform boundedness theorem.
22. If M and N are closed linear subspaces of Hilbert space H and P, Q are the projections on M and N,show that (i) M⊥N if and only if PQ=0 and (ii)PQ=0 if and only if QP=0.
23. With usual notations, prove that $r(x) = \lim \|x^n\|^{1/n}$.
24. State and prove Gelfand-Neumark theorem.

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