M.Sc. DEGREE EXAMINATION,NOVEMBER 2019
I Year I Semester
Real Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define a Measurable function.
2. Define Borel Function.
3. Define Positive and Negative parts of a real function.
4. Give an example of a function, which is measurable but not Riemann integrable.
5. Define Uniform convergence.
6. Define equi – continuous.
7. State inverse function theorem.
8. Define Contraction Principle
9. Define Power Series.
10. Define Gamma Function.
11. Define Simple Function.
12. Prove that constant function are measurable.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Show that there exists a non-measurable set.
14. State and Prove that Riemann Lebesgue Lamma.
15. If $\{\mathrm{F}_n\}$ is a sequence of continuous function on E and if fn $\rightarrow$ f uniformly on E, then prove that f is continuous on E.
16. If X is a complete metric space and if Φ is a contraction of X into X then prove that there exists one and only one x such than Φ(x)=x.
17. If f is continuous (with period 2π) and if $\in > 0$ then prove that there is a trigonometry polynomial P such that $|p(x) - f(x)| < \epsilon$, for all real x.
18. Let f, g be non-negative measurable functions show that
$$\int (f+g)dx = \int f dx + \int g dx$$

19. Prove that every intervals are measurable.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Show that the outer measure of an interval equals its length
21. State and prove Fatou's Lemma.
22. State and prove stone – weierstrass Theorem.
23. State and prove implicit function theorem.
24. If f is positive function on (0, ∞) such that

    (a) f(x+1) = xf(x)

    (b) f(1) = 1

    (c) log f is convex

    Then prove that f(x)= $\Gamma(x)$