M.Sc. DEGREE EXAMINATION,NOVEMBER 2019
II Year III Semester
Complex Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Find the zeros of $\mathbf{f}\left(\mathbf{z}\right)=\mathbf{c}os\left(\frac{\mathbf{1+z}}{\mathbf{1-z}}\right),\ \ |\mathbf{z}|<\mathbf{1}$.
2. Define starshaped set.
3. State open mapping theorem.
4. Find the poles of $\mathbf{f}\left(\mathbf{z}\right)=\left(\mathbf{1-e^{z}}\right)^{-\mathbf{1}}$.
5. Define meromorphic function.
6. Define infinite product of sequence of complex numbers.
7. Prove that $\mathbf{l}im_{\mathbf{z}\to\mathbf{0}}\frac{\mathbf{l}og(\mathbf{1+z})}{\mathbf{z}}=\mathbf{1}$.
8. Prove $\mathbf{\Gamma}\left(\mathbf{z+1}\right)=\mathbf{z\Gamma}\left(\mathbf{z}\right)$.
9. State mean value property.
10. Define Green's function.
11. Write Poisson-Jenson formula.
12. Define Landau's constant.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. State and prove fundamental theorem of algebra.
14. If *G* is simply connected and $\mathbf{f}:\mathbf{G}\to\mathbb{C}$ is analytic then prove that *f* has a primitive in *G*.
15. State and prove Casorati-Weierstrass theorem.
16. If $|\mathbf{z}|\leq\mathbf{1}$ and $\mathbf{p}>\mathbf{0}$ prove that $|\mathbf{1-E_p}\left(\mathbf{z}\right)|\leq|\mathbf{z}|^{\mathbf{p+1}}$.
17. Prove that $\left\{\left(\mathbf{1+\frac{z}{n}}\right)^{\mathbf{n}}\right\}$ $\mathbf{c}onvegers\ \mathbf{t}o\ \mathbf{e^z}\ \mathbf{i}n\ \mathbf{H}(\mathbb{C})$.
18. Let *G* be a region and suppose that *u* is a continuous real valued function on G with the MVP. If there is a appoint *a* in G such that $\mathbf{u(a)}\geq\mathbf{u}\left(\mathbf{z}\right)\ \mathbf{f}or\ \mathbf{a}ll\ \mathbf{z}\in\mathbf{G}$, prove that *u* is a constant function.

19. Let f be an entire function of finite order, then prove that f assumes each complex number with one possible exception.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Let G be an open set and let $\mathbf{f}:\mathbf{G} \rightarrow \mathbb{C}$ be a differentiable function prove that f is analytic on G.
21. Show that $\int_{\mathbf{0}}^{\infty} \frac{\mathbf{x}^{-\mathbf{c}}}{\mathbf{1}+\mathbf{x}} \mathbf{d}x = \frac{\pi}{\mathbf{s}in\pi\mathbf{C}}$, $\mathbf{i}f\ \mathbf{0}<\mathbf{c}<\mathbf{1}$.
22. Let G be a region which is not the whole plane and such that every non-vanishing analytic function on G has an analytic square root. If $\mathbf{a} \in \mathbf{G}$ prove that there is an analytic function f on G such that

    (i) $\mathbf{f}(\mathbf{a}) = \mathbf{0}$ $\mathbf{a}nd$ $\mathbf{f}^{'}(\mathbf{a}) > \mathbf{0}$;

    (ii) $\mathbf{f}$ $\mathbf{i}s$ $\mathbf{o}ne-\mathbf{o}ne$;

    (iii) $\mathbf{f}(\mathbf{G}) = \mathbf{D} = \{\mathbf{z}:|\mathbf{z}| < \mathbf{1}\}$.
23. State and prove Harnack's theorem.
24. If f is an entire function of finite order λ, then prove that f has finite genus $\mu \leq \lambda$.

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