# B.Sc. DEGREE EXAMINATION,NOVEMBER 2019
## I Year II Semester
## Classical Algebra

**Time : 3 Hours** **Max.marks :75**

### Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Find the coefficient of$x^6$ in the expansion of $\frac{1}{(1-x^2)^3}$.
2. If $y = x - \frac{x^2}{2} + \frac{x^3}{3} - ...$, prove that $x = y + \frac{y^2}{2!} + \frac{y^3}{3!} - ...$
3. Find the value of *k* if $3 + 2i$ is a root of the equation$x^2 - 6x + k = 0$.
4. If $\alpha$, $\beta$, $\gamma$ are the roots of the equation$x^3 - 7x + 6 = 0$, find $\sum \frac{1}{\alpha}$.
5. Find the equation whose roots are the root of the equation $\mathbf{4x^4+32x^3+83x^2+76x+21=0}$ increased by 2.
6. Define reciprocal equation.
7. State Cayley-Hamilton theorem.
8. Show that the diagonal elements of a skew symmetric matrix are zero.
9. Find the eigen values of $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.
10. Define Euler function $\phi(\mathbf{N})$.
11. Show that for any integer *n*, $\mathbf{n(n+1)(2n+1)}$ is divisible by 6.
12. Find the number of divisors of 360.

### Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Sum to infinity the series $1 + \frac{2^4}{\underline{2}} + \frac{3^4}{\underline{3}} + \frac{4^4}{\underline{4}} + ....\infty$
14. Solve the equation $x^3 - 19x^2 + 114x - 216 = 0$ given that the roots are in Geometric Progression.
15. Solve the equation $6x^4 - 13x^3 - 35x^2 - x + 3 = 0$one of whose roots is $2 - \sqrt{3}$.
16. Solve the equation $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

17. Show that the matrix $\begin{pmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{pmatrix}$ is unitary.
18. Find the Eigen values of $\begin{bmatrix} \mathbf{-2} & \mathbf{2} & \mathbf{-3} \\ \mathbf{2} & \mathbf{1} & \mathbf{-6} \\ \mathbf{-1} & \mathbf{-2} & \mathbf{0} \end{bmatrix}$
19. With how many zeros does $\mathbf{79!}$ end?

## **Section C** $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Find the sum to infinity of the series $\mathbf{1}+\frac{2}{6}+\frac{2.5}{6.12}+\frac{2.5.8}{6.12.18}+\ldots$
21. If $\propto, \beta, \gamma$ are the roots of $\mathbf{x^3}+p\mathbf{x^2}+q\mathbf{x}+r=\mathbf{0}$ find (i) $\sum \propto^{\mathbf{2}}$, (ii) $\sum \propto^{\mathbf{2}}\beta$ and (iii) $\sum \propto^{\mathbf{2}}\beta^{\mathbf{2}}$
22. Solve: $3x^6+x^5-27x^4+27x^2-x-3=0$.
23. Verify Cayley-Hamilton theorem for $\mathbf{A}=\begin{pmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{1} \\ \mathbf{-1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{1} & \mathbf{-1} & \mathbf{2} \end{pmatrix}$.
24. State and Prove Fermat's theorem.

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