B.Sc. DEGREE EXAMINATION,NOVEMBER 2019
I Year II Semester
Integral Calculus and Fourier Series

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Evaluate $\int \mathrm{x}^2 \sin \mathrm{x}\, \mathrm{dx}$
2. Evaluate $\int \mathrm{x}^3 \mathrm{e}^{-2\mathrm{x}} \mathrm{dx}$
3. Define double integral
4. Evaluate $\int_0^a \int_0^b \left(x^2 + y^2\right) dx\ dy$
5. Evaluate $\int_0^{\frac{\pi}{2}} sin^7\theta\ \ cos^5\ d\theta$
6. State any two properties of Beta functions
7. Find the constant a_0 of the Fourier series for the function $f\left(x\right) = k$ in $0 \leq x \leq 2\pi$
8. Find the Fourier constants b_n for $x\ sin\ x$ in $(-\pi,\ \pi)$
9. Find a sine series for $f\left(x\right) = c$ in the range 0 to π.
10. If $f\left(x\right) = x^2$ in $-\pi \leq x \leq \pi$, find the value a_0 of the fourier series.
11. Evaluate $\int_0^{\infty} e^{-x^2} dx$
12. Define triple integral

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Evaluate $\int e^x\ sin\ 3x\ cos\ 2x\ dx$
14. Evaluate $\iint (x^2 + y^2)\ dx\ dy$ over the region for which $x,\ y$ are each ≥ 0 and $x + y \leq 1$
15. If $n > 0$, prove that $\Gamma(\mathrm{n}+1) = \mathrm{n}\ \Gamma(\mathrm{n})$
16. Find the Fourier series to represent $x - \pi$ in the interval $(-\pi,\ \pi)$

17. Find a Fourier series with period 3 to represent $f\left(x\right)=2x-x^{3}$ in the range $\left(0,3\right)$

18. Find a sine series for

$$f(x)=\begin{cases} x & when\ 0<x<\frac{\pi}{2} \\ 0 & when\ \frac{\pi}{2}<x<\pi \end{cases}$$

19. Prove that $\Gamma\left(\dfrac{1}{2}\right)=\sqrt{\pi}$

Section C $(3\times 10=30)$ Marks

Answer any **THREE** questions

20. If $I_{m,n}=\displaystyle\int x^{m}\left(log\ x\right)^{n}dx$ (where m and n are positive integers), show that $I_{m,n}=\left(log\ x\right)^{n}\dfrac{x^{m+1}}{m+1}-\dfrac{n}{m+1}\ I_{m,n-1}$. Hence find $\displaystyle\int x^{4}(log\ x)^{3}dx$

21. Evaluate $\displaystyle\iiint_{V}\left(x+y+z\right)\,dx\ dy\ dz$, where the region V is bounded by $x+y+z=a\quad(a>0)$, $x=0,\ y=0,\ z=0.$

22. Express $\displaystyle\int_{0}^{1}x^{m}(1-x^{n})^{p}dx$ in terms of Gamma functions and evaluate the integral $\displaystyle\int_{0}^{1}x^{5}\left(1-x^{3}\right)^{10}dx$

23. Show that $\mathbf{x^{2}}=\dfrac{\pi^{\mathbf{2}}}{\mathbf{3}}+\mathbf{4}\displaystyle\sum_{\mathbf{n=1}}^{\infty}\left(-\mathbf{1}\right)^{\mathbf{n}}\dfrac{\mathbf{c}osnx}{\mathbf{n^{2}}}$ in the interval $\left(-\pi\leq\mathbf{x}\leq\pi\right)$

24. If the function $\mathbf{y}=\mathbf{x}$ in the range 0 to π is expanded as a sine series, show that it is equal to $\mathbf{2}\left(\dfrac{\mathbf{s}in\ \mathbf{x}}{\mathbf{1}}-\dfrac{\mathbf{s}in\ \mathbf{2x}}{\mathbf{2}}+\dfrac{\mathbf{s}in\ \mathbf{3x}}{\mathbf{3}}\ldots\right)$

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