B.Sc. DEGREE EXAMINATION,NOVEMBER 2019
II Year IV Semester
Vector Calculus and Fourier Transforms

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. If $\emptyset = \mathbf{x^2y+y^2x+z^2}$ find $grad\emptyset$ at (1, 1, 1).
2. Determine the constants '*a*' so that the vector $\overrightarrow{\mathbf{F}} = (\mathbf{z+3y})\ \overrightarrow{\mathbf{i}} + (\mathbf{x-2z})\ \overrightarrow{\mathbf{j}} + (\mathbf{x+a}z)\ \overrightarrow{\mathbf{k}}$ is solenoidal.
3. If $\overrightarrow{\mathbf{A}}$ $\mathbf{a}nd$ $\overrightarrow{\mathbf{B}}$ are irrotational. Show that $\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}$ is solenoidal.
4. If $\overrightarrow{F} = \left(3t^2 - 1\right)\widehat{i} + (2 - 6t)\widehat{j} - 4t\widehat{k}$, find $\int_2^3 \overrightarrow{F}\,dt$.
5. State Green's theorem
6. Show that $\overrightarrow{\mathbf{F}} = \left(\mathbf{2x}y + \mathbf{z^3}\right)\ \overrightarrow{\mathbf{i}} + \mathbf{x^2}\ \overrightarrow{\mathbf{j}} + \mathbf{3xz^2}\ \ \overrightarrow{\mathbf{k}}$ is a conservative field.
7. If $\overrightarrow{\mathbf{F}} = \mathbf{3x}y\ \overrightarrow{\mathbf{i}} - \mathbf{y^2}\ \overrightarrow{\mathbf{j}}$ and C is a curve $\mathbf{y=2x^2}$ joining the point $(\mathbf{0}, \mathbf{0})$ $\mathbf{t}o$ $(\mathbf{1}, \ \mathbf{2})$, then find $\int_{\mathbf{C}} \overrightarrow{\mathbf{F}}.\mathbf{d}\overrightarrow{\mathbf{r}}$.
8. State Gauss Divergence theorem.
9. State the convolution theorem for Fourier transforms
10. Find the Fourier sine transform of $\frac{\mathbf{1}}{\mathbf{x}}$.
11. Define Parseval's Identity
12. State Fourier integral theorem.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Find the unit vector normal to the surface $\mathbf{x^2+3y^2+2z^2=6}$ at the point (2, 0, 1).
14. If $\overrightarrow{\mathbf{r}} = \mathbf{x}\,\overrightarrow{\mathbf{i}} + \mathbf{y}\,\overrightarrow{\mathbf{j}} + \mathbf{z}\,\overrightarrow{\mathbf{k}}$, then prove that $\nabla \times \mathbf{f}\,(\mathbf{r})\,\overrightarrow{\mathbf{r}} = \overrightarrow{\mathbf{0}}$
15. If $\overrightarrow{\mathbf{F}} = \left(\mathbf{3x^2+6y}\right)\ \overrightarrow{\mathbf{i}} - \mathbf{14y}z\,\overrightarrow{\mathbf{j}} + \mathbf{20xz^2}\,\overrightarrow{\mathbf{k}}$. Evaluate $\int_{\mathbf{C}} \overrightarrow{\mathbf{F}}\ .\mathbf{d}\,\overrightarrow{\mathbf{r}}$ from (0,0,0) to (1,1,1) along the path $\mathbf{x=t}$, $\mathbf{y=t^2}$, $\mathbf{z=t^3}$.
16. Verify Green's theorem in the plane for $\int_{\mathbf{C}} \left(\mathbf{x}y + \mathbf{y^2}\right)\mathbf{d}x + \mathbf{x^2}\mathbf{d}y$ where C is the closed curve of the region bounded by $\mathbf{y=x^2}$ and $\mathbf{y=x}$
17. Verify Stoke's theorem for the function $\overrightarrow{\mathbf{F}} = \mathbf{x^2}\,\overrightarrow{\mathbf{i}} + \mathbf{x}y\,\overrightarrow{\mathbf{j}}$ over the square with sides $x = 0$, $x = a$, $y = 0$, $y = a$ in the plane $z = 0$

18. Find the Fourier transform of $\mathbf{f}\left(\mathbf{x}\right)=\begin{cases}\mathbf{x} & |\mathbf{x}|<\mathbf{a}\\ \mathbf{0} & |\mathbf{x}|>\mathbf{a}\end{cases}$
19. Find the Fourier cosine transform of $\mathbf{f}\left(\mathbf{x}\right)=\frac{\mathbf{1}}{\mathbf{1+x^2}}$

Section C $(3\times 10=30)$ Marks

Answer any **THREE** questions

20. (i) Prove that $\mathbf{c}url\left(\mathbf{g}rad\emptyset\right)=\mathbf{0}$.

    (ii) Prove that $\nabla.\left(\mathbf{r^n}\overrightarrow{\mathbf{r}}\right)=\left(\mathbf{n+3}\right)\mathbf{r^n}$.
21. Evaluate $\iint_{\mathbf{s}}\overrightarrow{\mathbf{F}}.\overrightarrow{\mathbf{n}}\mathbf{d}s$ where $\overrightarrow{\mathbf{F}}=\mathbf{4x}z\,\overrightarrow{\mathbf{i}}-\mathbf{y^2}\,\overrightarrow{\mathbf{j}}+\mathbf{y}z\,\overrightarrow{\mathbf{k}}$ and S is the surface of the cube bounded by $\mathbf{x=0,y=0,z=0,x=1,y=1}$ and $\mathbf{z=1}$
22. Verify the divergence theorem for $\overrightarrow{\mathbf{F}}=\mathbf{y}\,\overrightarrow{\mathbf{i}}+\mathbf{x}\,\overrightarrow{\mathbf{j}}+\mathbf{z^2}\,\overrightarrow{\mathbf{k}}$ over the cylindrical region bounded by $\mathbf{x^2+y^2=a^2},\ \mathbf{z=0},\ \mathbf{z=h}$
23. Find the Fourier transform of $\mathbf{f}\left(\mathbf{x}\right)=\begin{cases}\mathbf{1-x^2} & |\mathbf{x}|\leq\mathbf{1}\\ \mathbf{0} & |\mathbf{x}|>\mathbf{1}\end{cases}$ and hence evaluate $\int_{\mathbf{0}}^{\infty}\left(\frac{\mathbf{x}cosx-\mathbf{s}inx}{\mathbf{x^3}}\right)\mathbf{c}os\frac{\mathbf{x}}{\mathbf{2}}.\mathbf{d}x$
24. Using Parseval's identity show that

    (i) $\int_0^{\infty}\frac{dx}{\left(x^2+1\right)^2}=\frac{\pi}{4}$

    (ii) $\int_0^{\infty}\frac{x^2dx}{\left(x^2+1\right)^2}=\frac{\pi}{4}$

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