B.Sc. DEGREE EXAMINATION,NOVEMBER 2019
III Year V Semester
Modern Algebra

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Show that Union of 2 subgroups is a subgroup.
2. Is the converse of Lagrange's theorem is true? Justify.
3. Define an Automorphism of a group. Give an example.
4. If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$ find $(\alpha\beta)^{-1}$
5. Show that any unit in a ring R, cannot be a zero divisor.
6. What is the characteristic value of ring of integer Z.
7. When an integral domain R is said to be a Euclidean ring?
8. Define Domain of Gaussian Integers.
9. When a polynomial p(x) is said to be irreducible over F.
10. Define primitive polynomial.
11. Prove that kernel $f = \{e\}$ ⇔f is one-one.
12. If R is a ring such that $a^2 = a$for all $a \in R$, then prove that $a + a = 0$

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Show that if H and K are finite subgroups of G of $O(H)$ and $O(K)$ respectively then $O\left(HK\right) = \frac{O(H)O(K)}{O(H \cap K)}$
14. Show that $I(G) \simeq \frac{G}{Z}$, where I(G) is the group of inner authomorphism of G and Z is the center of G.
15. Let R be a commutative ring with identity. Then prove that R is a field if and only if R has not proper ideals.
16. Let R be a Euclidean ring suppose that for $a, b, c \in R$, $a|bc$ but $(a, b) = 1$ then prove that a/c

17. State and prove the Eisentein criteria of irreducible of polynomial.
18. State and prove Lagrange's theorem.
19. Let R be a Euclidean ring and If $b \neq 0$ is not a unit in R then prove that $d(a) < d(ab)$

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Let φ be a homomorphism of G onto G' with kernel K. Show that

    (i) K is normal subgroup of G

    (ii) $\frac{G}{K} \simeq G'$
21. Prove that every group is isomorphic to a subgroup of A(S) for some appropriate S
22. If R is a commutative ring, with unit element and M is an ideal of R, prove that M is a maximal ideal of R if and only if ${}^{R}/_{M}$ is a field.
23. State and prove unique factorization theorem.
24. State and prove division Algorithm of polynomials.

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