B.Sc. DEGREE EXAMINATION,NOVEMBER 2019
III Year V Semester
Real Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Justify : Z is countable.
2. Give an example of Countable bounded subset A of R whose glb and lub are both in R-A
3. Find $\lim\limits_{n\to\infty} \; inf(-1)^n$ and $\lim\limits_{n\to\infty} \; sup(-1)^n$.
4. When a series $\sum_{n=1}^{\infty} a_n$ is said to be converges conditionally.
5. Check $f(x) = \frac{\sin x}{x}, (x \in R^1, x \neq 0)$ is continuous or not?
6. Prove $\lim\limits_{x\to 3} \; (x^2 + 2x) = 15$.
7. Is every bounded set is totally bounded. Justify.
8. Define contraction mapping.
9. Find a suitable 'c' of Rolle's theorem for the function $f(x) = (x-a)(b-x), a \leq x \leq b$
10. State mean value theorem.
11. Justify : A divergent sequence may have a convergent sub sequence.
12. If $\sum_{n=1}^{\infty} a_n$ is a convergent series then prove that $\lim\limits_{n\to\infty} \; a_n = 0$

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove : Countable union of countable sets is countable
14. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.
15. Prove that the continuous function of a continuous function is continuous.
16. If the subset A of the metric space (M, ρ) is totally bounded, then prove that A is bounded.
17. State and prove second fundamental theorem of calculus.
18. State and prove Ratio test.

19. Prove that f is continuous if and only if the inverse image of every open set is open.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that a increasing sequence which is bounded above is convergent. Hence, deduce $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$is convergent.
21. Prove that the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is converges if and only if it is Cauchy.
22. Let (M, ρ) be a metric space and 'a' be a point in M. Let f and g are real valued functions whose domains are subsets of M. If $(\lim_{x\to a} \ f(x)) = L$ and $\lim_{x\to a} \ g(x) = N$, then prove that

    (i) $\lim_{x\to a} \ [f(x)g(x)] = LN$

    (ii)$\lim_{x\to a} \ \frac{f(x)}{g(x)} = \frac{L}{N}, if N \neq 0.$
23. Prove that A of R' is connected if and only if whenever $a \in A, b \in A$ with $a < b$ then $c \in A$ for any c such that $a < c < b$.
24. State and prove Chain rule of derivatives.

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