B.Sc. DEGREE EXAMINATION,NOVEMBER 2019
III Year VI Semester
Complex Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Continuity of a function at a point.
2. Define analytic function.
3. Define a simple closed curve.
4. State Cauchy –Goursat theorem
5. Evaluate the integral $\int_C \overline{z}dz$ where C is the upper half of the circle $|z| = 1$.
6. State Liouville's theorem.
7. Evaluate $\int_C \frac{z\ dz}{z-2}$ where C is the circle $|z| = 1$.
8. State Laurent's series.
9. Find the residue of the function $f(z) = \frac{z}{z^2+1}$ at its poles.
10. Find the singular points of the function $f(z) = \frac{1}{z(z-i)}$
11. Define Bilinear transformation
12. Find the fixed points of the function $w = \frac{6z-9}{z}$

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove that f(z) is analytic only if the partial derivatives exist and satisfy the cauchy- Riemann equations .
14. Find the analytic function $f(z) = u + iv$ of which the real part is $u = e^x(x\ cosy - y\ siny)$
15. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$.
16. Show that $\left| \int_C \frac{z+4}{z^3-1} dz \right| \leq \frac{6\pi}{7}$, Where C is the arc of the circle $|z| = 2$ from z =2 to z= 2i

17. Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor's series about the point z=1 and determine the region of convergence.
18. Determine and classify the singular points for the function $f(z) = \frac{z}{e^z - 1}$.
19. Find the bilinear transformation which maps the points z = -2,0,2 in to the points w = 0,i, -i respectively.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Show that the function $u = \frac{1}{2}\log\left(x^2 + y^2\right)$ is harmonic and determine its conjugate.
21. State and prove Cauchy integral formula.
22. If f(z) $= \frac{z+4}{(z+3)(z-1)^2}$ find the Laurent's series expansions in (i) $0 < |z-1| < 4$ (ii) $|z-1| > 4$
23. If C is the unit circle about the origin described in the positive sense, show that $(i) \int_C \frac{e^{-z}}{z^2}\, dz = 2\pi i$ (ii) $\int_C \frac{dz}{z\ sinz} = 0.$
24. Discuss the transformation $w = \frac{1}{z}$.

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