

**B.Sc.DEGREE EXAMINATION, APRIL 2020**  
**I Year I Semester**  
**Differential Calculus**

**Time : 3 Hours**

**Max.marks :75**

**Section A** ( $10 \times 2 = 20$ ) Marks

Answer any **TEN** questions

1. If  $y = \log(ax+b)$ . Find  $y_n$ .
2. Find the  $n$ th derivative of  $e^{ax}$ .
3. If  $x = r \cos \theta$ ,  $y = r \sin \theta$  find  $\frac{\partial(x, y)}{\partial(r, \theta)}$
4. Find the minimum value of  $x^3 + 5y^2 - 6x + 10y + 12$ .
5. Write down the cartesian formula for radius of curvature.
6. Find the radius of curvature at the point  $(\frac{a}{4}, \frac{a}{4})$  to the curve  $\sqrt{X} + \sqrt{Y} = a$ .
7. Show that the radius of curvature at the point  $\theta$  on the curve  $x = a(\cos \theta + \theta \sin \theta)$ ,  $y = a(\sin \theta - \theta \cos \theta)$  is  $a\theta$ .
8. Find the (p-r) equation for the curve  $r \sin \theta + a = 0$ .
9. Define an asymptote of a curve.
10. State Leibnitz's theorem.
11. Find the asymptote of the curve  $x^3 + y^3 = 3axy$ .
12. State the method of Lagrange's Multipliers.

**Section B** ( $5 \times 5 = 25$ ) Marks

Answer any **FIVE** questions

13. Find the  $n^{th}$  derivative of  $\frac{2x+1}{(2x-1)(2x+3)}$ .
14. Show that the maximum value of  $x^2 y^2 z^2$  subject to the constraint  $x^2 + y^2 + z^2 = a^2$  is  $(\frac{a^2}{3})^3$
15. Calculate the first derivative of the curve  $x^3 + y^3 = 3axy$  at the point  $x = y = \frac{3a}{2}$
16. Find the radius of curvature of the curve  $r^2 = a^2 \cos 2\theta$
17. Find the asymptote of the curve  $y^3 - 2y^2 x - yx^2 + 2x^3 + x^2 - 6xy + 5y^2 - 2y + 2x + 1 = 0$ .
18. Find the  $n$ th derivative of  $e^x \log x$ .
19. If  $u = \frac{1}{x}$ ,  $v = \frac{x^2}{y}$ ,  $w = x + y + zy^2$  find  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ .

**Section C** ( $3 \times 10 = 30$ ) MarksAnswer any **THREE** questions

20. If  $y = (\sin^{-1}x)^2$  Prove that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$ . Hence find the value of  $y_n$  at  $x = 0$ .
21. Find the minimum of  $a^3x^2 + b^3y^2 + c^3z^2$  with the condition  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ .
22. Find the coordinates of the centre of curvature at the point  $(2at, at^2)$  of the curve  $x^2 = 4ay$
23. Find the pedal equation of the curve  $r = a(1 - \cos\theta)$ .
24. Find the asymptotes of the curve  $4(x^4 + y^4) - 17x^2y^2 - 4x(4y^2 - x^2) + 2(x^2 - 2) = 0$  and show that they pass through eight points of intersection of the curve with an ellipse.

**B.Sc.DEGREE EXAMINATION, APRIL 2020**  
**I Year I Semester**  
**Differential Calculus**

**Time : 3 Hours**

**Max.marks :75**

**Section A** ( $10 \times 2 = 20$ ) Marks

Answer any **TEN** questions

1. If  $y = \log(ax+b)$ . Find  $y_n$ .
2. Find the  $n$ th derivative of  $e^{ax}$ .
3. If  $x = r \cos \theta$ ,  $y = r \sin \theta$  find  $\frac{\partial(x, y)}{\partial(r, \theta)}$
4. Find the minimum value of  $x^3 + 5y^2 - 6x + 10y + 12$ .
5. Write down the cartesian formula for radius of curvature.
6. Find the radius of curvature at the point  $(\frac{a}{4}, \frac{a}{4})$  to the curve  $\sqrt{X} + \sqrt{Y} = a$ .
7. Show that the radius of curvature at the point  $\theta$  on the curve  $x = a(\cos \theta + \theta \sin \theta)$ ,  $y = a(\sin \theta - \theta \cos \theta)$  is  $a\theta$ .
8. Find the (p-r) equation for the curve  $r \sin \theta + a = 0$ .
9. Define an asymptote of a curve.
10. State Leibnitz's theorem.
11. Find the asymptote of the curve  $x^3 + y^3 = 3axy$ .
12. State the method of Lagrange's Multipliers.

**Section B** ( $5 \times 5 = 25$ ) Marks

Answer any **FIVE** questions

13. Find the  $n^{th}$  derivative of  $\frac{2x+1}{(2x-1)(2x+3)}$ .
14. Show that the maximum value of  $x^2 y^2 z^2$  subject to the constraint  $x^2 + y^2 + z^2 = a^2$  is  $(\frac{a^2}{3})^3$
15. Calculate the first derivative of the curve  $x^3 + y^3 = 3axy$  at the point  $x = y = \frac{3a}{2}$
16. Find the radius of curvature of the curve  $r^2 = a^2 \cos 2\theta$
17. Find the asymptote of the curve  $y^3 - 2y^2 x - yx^2 + 2x^3 + x^2 - 6xy + 5y^2 - 2y + 2x + 1 = 0$ .
18. Find the  $n$ th derivative of  $e^x \log x$ .
19. If  $u = \frac{1}{x}$ ,  $v = \frac{x^2}{y}$ ,  $w = x + y + zy^2$  find  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ .

**Section C** ( $3 \times 10 = 30$ ) MarksAnswer any **THREE** questions

20. If  $y = (\sin^{-1}x)^2$  Prove that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$ . Hence find the value of  $y_n$  at  $x = 0$ .
21. Find the minimum of  $a^3x^2 + b^3y^2 + c^3z^2$  with the condition  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ .
22. Find the coordinates of the centre of curvature at the point  $(2at, at^2)$  of the curve  $x^2 = 4ay$
23. Find the pedal equation of the curve  $r = a(1 - \cos\theta)$ .
24. Find the asymptotes of the curve  $4(x^4 + y^4) - 17x^2y^2 - 4x(4y^2 - x^2) + 2(x^2 - 2) = 0$  and show that they pass through eight points of intersection of the curve with an ellipse.