Bsc. DEGREE EXAMINATION,APRIL 2020
I Year II Semester
Integral Calculus and Fourier Series

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Evaluate $\int x^3 \ e^x$ dx.
2. Evaluate $\int cos^n x$ dx.
3. Evaluate $\int_0^1 \int_0^2 (x^2 \ +y^2 \ )$dy dx.
4. Evaluate $\int\int xydxdy$ over the region in the positive quadrant for which x+y =1.
5. Prove that $\Gamma(1)$=1
6. Evaluate $\int_0^1 (xlogx)^4$ dx.
7. Find the Fourier coefficient a_0 for the function f(x)= $\frac{(\pi - x)}{2} \quad for \ all \ 0<x<2\pi.$
8. Define Fourier series of a function f(x) in the interval 0 to 2 π.
9. Define Fourier cosine series of a function f(x) in the interval 0 to π .
10. State Dirichlet's condition.
11. State Bernoulli's formula for integration.
12. Evaluate $\int_0^1 x^7 \ (1-x)^3$ dx.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Evaluate $\int (logx)^3 \ x^4$ dx
14. Evaluate $\int cos3x \ e^{2x}$ dx.
15. Find by double integration the area between the parabola $y = 4x - x^2 andtheline$y=x.
16. Prove that $\Gamma\frac{1}{2} = \sqrt{\pi}$ using gamma functions .
17. Find a fourier series for $f(x) = \begin{cases} 1, & -\pi \le x \le 0 \\ 2, & 0 \le x \le \pi \end{cases}$
18. Prove that $\beta \ (m,n) = \beta \ (m, n+1) + \beta(m+1, n)$
19. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}$ dxdydz.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If $I_n = \int_0^1 x^p (1-x)^n \ dx$, where p , q $>$0 and n is a positive integer. Prove that (P+qn+1)I_n =nq$I_{n-1}$and hence evaluate I_4.
21. Evaluate by changing the order of integration of $\int_1^{\mathbf{2}} \int_0^{4-x^2} (x+y)\, dydx$
22. Obtain the relation between Beta and Gamma function.
23. Obtain a Fourier series expansion for e^x in the interval $-\pi < x < \pi$
24. Obtain the half range cosine series for f(x)=x in the interval $0 < x < \pi$ and Deduce that the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots.. \infty.?$