B.Sc.DEGREE EXAMINATION,APRIL 2020
II Year III Semester
Allied Mathematics-I

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Write down the expansion for $\dfrac{1}{(1-x)^n}$.
2. Show that $\dfrac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots\ldots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots\ldots} = \frac{e-1}{e+1}$.
3. State the Cayley-Hamilton theorem.
4. If the sum of two eigen values and trace of a 3×3 matrix A are equal, find the value of |A|.
5. Express tan $n\theta$ in terms of powers of tanθ.
6. Find $\lim_{x \to 0} \dfrac{x - \sin x}{x^3}$.
7. Show that $E = 1 + \Delta$.
8. State Newton's Backward difference interpolation formula.
9. Prove that $\cosh^2 x - \sinh^2 x = 1$
10. If $\cosh x = \sec\alpha$, then find $\sinh x$ and $\tanh x$
11. Prove that $e^{i\,\theta} = \cos\theta + i\sin\theta$ for all θ
12. Write the expansion of $\log(1 + x)$.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Show that $\displaystyle\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!} = \frac{e}{2} + \frac{2}{e}$.
14. Prove that the given matrix A = $\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is orthogonal.
15. Prove that $\dfrac{\sin 7\theta}{\sin\theta} = 7 - 56\sin^2\theta + 112\sin^4\theta - 64\sin^6\theta$.

16. Using Newton's forward interpolation formula find the value of y at $x = 28$.

x	20	23	26	29
y	0.3420	0.3907	0.4384	0.4848

17. Separate into real and imaginary parts of $\operatorname{sec}h(x - iy)$.

18. Find the eigen values and eigenvector of A = $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$.

19. Find the sum to infinity the series $1 + \dfrac{1+5}{2!} + \frac{1+5+5^2}{3!} + ...$

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that $\dfrac{1}{1.2.3} + \frac{1}{3.4.5} + \frac{1}{5.6.7} + ... = \log 2 - \frac{1}{2}$.

21. Verify Cayley – Hamilton theorem for the matrix A = $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 0 & 0 \\ 0 & -2 & 1 \end{pmatrix}$. Also find A^{-1}.

22. Show that $2^{11}\sin^5\theta\cos^7\theta = \sin 12\theta + 2\sin 10\theta - 4\sin 8\theta - 10\sin 6\theta + 5\sin 4\theta + 20\sin 2\theta$.

23. Given : $\log_{10} 300 = 2.4771$, $\log_{10} 304 = 2.4829$, $\log_{10} 305 = 2.4843$, $\log_{10} 307 = 2.4871$. Using Lagrange's formula find $\log_{10} 301$.

24. If $\tan(x + iy) = u + iv$ then prove that $\dfrac{u}{v} = \frac{\sin 2x}{\sinh 2y}$.

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