B.Sc.DEGREE EXAMINATION, APRIL 2020 II Year IV Semester Vector Calculus and Fourier Transforms

Time : 3 Hours

Max.marks:75

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

- 1. Define Unit Normal Vector.
- 2. Define Solenoidal Vector.
- 3. Define Line Integral.
- 4. If $\overrightarrow{F} = 3xy \overrightarrow{i} y^2 \overrightarrow{j}$ evaluate $\int_c \overrightarrow{F} d\overrightarrow{r}$ where C is the curve on the xy plane $y = 2x^2$ from (0,0) to (1,2)
- 5. State Gauss Theorem.
- 6. Evaluate by Stoke's theorem, $\int_c \left(e^x dx + 2y dy dz\right)$ where C is the curve $x^2 + y^2 = 4^2, z = 2$
- 7. State Linearity Property.
- 8. Define Dirichlet's Condition.
- 9. Define Parseval's Identity.
- 10. State Unit step function.

11. If $\overrightarrow{r} = t^2 \overrightarrow{i} - t \overrightarrow{j} + (2t+1) \overrightarrow{k}$ then find the velocity at t=0.

12. Define Fourier Transforms.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

- 13. Find the unit tangent vector to the curve x = 3t + 2, $y = 5t^2$, z = 2t 1 at the point where t = 1.
- 14. Prove that $\nabla \mathbf{r}^n = \mathbf{n} r^{n-2} \overrightarrow{r}$ where \overrightarrow{r} is the position vector.
- 15. If $\overrightarrow{F} = yz \overrightarrow{i} + zx \overrightarrow{j} + xy \overrightarrow{k}$, find $\int_c \overrightarrow{F} d\overrightarrow{r}$ where c is given by $x = t, \ y = t^2 \cdot z = t^3$ from P (0,0,0) to Q (2,4,8).
- 16. Find the Fourier cosine integral representation of the function $f(x) = xe^{-2x}$, x > 0.
- 17. State and prove Fourier Transform of Unit impulse Function.

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- 18. Find $\int_c \overrightarrow{F} \cdot d\overrightarrow{r}$ where $\overrightarrow{F} = (x^2 y^2) \overrightarrow{i} + 2xy \overrightarrow{j}$ and c is a square bounded by the co-ordinate axes and the lines x = a and y = a
- 19. Prove that $\nabla \times \nabla \Phi = \vec{0}$

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

- 20. Prove that $\nabla^2 f(\vec{r}) = \left(\frac{\partial^2 f}{2} + 2\right)$
- 21. Find the total work done in moving a particle in a force field given by $\overrightarrow{F} = 3xy\overrightarrow{i} - 5z\overrightarrow{j} + 10x\overrightarrow{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from t =1 to t = 2.
- 22. Verify Gauss divergence theorem for $\overrightarrow{F} = (x^3 yz) \overrightarrow{i} 2x^2y \overrightarrow{j} + 2\overrightarrow{k}$, taken over the cube bounded by x = 0, y = 0, z = 0, x = a, y = a, z = a.
- 23. Find the Fourier transform of f(x) defined by $f(x) = \begin{cases} x^2, & |x| < 0 \\ 0, & |x| \ge 0 \end{cases}$
- 24. State and Prove Convolution Theorem.

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