B.Sc.DEGREE EXAMINATION,APRIL 2020
III Year V Semester
Modern Algebra

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define subgroup with an example.
2. Define normal subgroup
3. Define an associatative ring.
4. Define an even permutation.
5. Define homomorphism on rings.
6. Define an ideal.
7. Define an Euclidean ring.
8. Define prime element.
9. State Gauss' Lemma
10. Define primitive polynomial.
11. If R is a ring, then prove that for all a,b $\in$R
    i) a0=0a=0. ii) a(-b)=(-a)b=-(ab)
12. Define an integral domain.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.
14. State and prove Cayley's theorem.
15. Prove that a finite integral domain is a field.
16. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.
17. State and prove Unique Factorization theorem.
18. State and prove the division algorithm.
19. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If Hand K are finite subgroups of G of order $O(H)$ and $O(K)$ respectively then $O\left(HK\right) = \dfrac{O\left(H\right)O\left(K\right)}{O\left(H \cap K\right)}$
21. If G is a group, then prove that *A(G)*, the set of automorphism of *G* is a group.
22. If U is an ideal of the ring R, then prove that R/U is a ring
23. Prove that every integral domain can be imbedded in a field.
24. State and prove fundamental theorem of homomorphism for ring theory.

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