B.Sc.DEGREE EXAMINATION,APRIL 2020
III Year V Semester
Real Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. If B is a countable subset of the uncountable set A, then prove that A – B is uncountable.
2. Define Cantor set.
3. Define the limit of the sequence.
4. Define Cauchy sequence.
5. Define metric space.
6. If G_1 and G_2 are open subsets of the metric space M, then prove that $G_1 \cap G_2$ is open.
7. Define complete metric space.
8. Define totally bounded subset of a metric space.
9. State the Law of mean.
10. State second fundamental theorem of calculus.
11. Find limit superior and the limit inferior of the sequence $s_n{=}(-1)^n,\ n\ \in I.$
12. Prove that the set of all integers is countable.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If B is an infinite subset of the countable set A, prove that B is countable.
14. If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers, and if $lim_{n\to\infty}\, sup\ s_n{=}lim_{n\to\infty}\, inf\ s_n{=}L,$ $when\ L \in R$, prove that $\{s_n\}_{n=1}^{\infty}$ is convergent and $lim_{n\to\infty}\, s_n\ {=}L.$
15. If $\lim\limits_{x\to a} f\,(x)\,{=}L$ and $lim_{x\to a}\, g\,(x)\,{=}M$, prove that $\lim\limits_{x\to a}\,[f\,(x)\,{+}g\,(x)]\,{=}L{+}M.$
16. If the subset A of the metric space M is totally bounded, prove that A is bounded.
17. State and prove Rolle's theorem.

18. If the sequence of real numbers $\{\mathbf{s_n}\}_{\mathbf{n=1}}^{\infty}$ is convergent, prove that $\{\mathbf{s_n}\}_{\mathbf{n=1}}^{\infty}$ is bounded.
19. State and prove comparison test.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that the set $[0, 1] = \{x/0 \leq x \leq 1\}$ is uncountable.
21. State and prove nested interval theorem.
22. Let $< M_1,\, \rho_1>$ and $< M_2,\, \rho_2>$ be metric spaces and let $f\colon M_1 \to M_2$. Prove that *f* is continuous on M_1 if and only if $f^{-1}(G)$ is open in M_1 whenever *G* is open in M_2
23. Let $<M, \rho>$ be a metric space, prove that the subset *A* of *M* is totally bounded if and only if every sequence of points of *A* contains a Cauchy subsequence.
24. State and prove first fundamental theorem of calculus.