B.Sc.DEGREE EXAMINATION,APRIL 2020
III Year V Semester
Graph Theory

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define K-regular graph.
2. Define cut-vertex of a graph G.
3. Define Eulerian trail.
4. Define Hamiltonian path.
5. Define independent set of edges.
6. Define tree and give an example.
7. Define planar graph.
8. Define maximum planar.
9. Define k-vertex colouring.
10. Define k-edge colouring.
11. Define adjacency matrix.
12. Define connected graph.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Define induced sub graph, edge induced sub graph and spanning sub graph.
14. If G is a Hamiltonian graph, prove that $\omega(G{-}S) \leq |S|$, for every non-empty subset S of V(G).
15. Prove that a graph G is a tree if and only if every two vertices of G are connected by a unique path.
16. Prove that A graph is planar if and only if it contains no contraction of K_5 or $K_{3,3}$.
17. Prove that there exists a k-colouring of a graph G if and only if V(G) can be partitioned into k subsets $V_1,\ V_2, \ldots,\ V_k$ such that no two vertices in $V_i,\ i{=}1, 2, \ldots, k$ are adjacent.

18. Prove that a vertex v in a connected graph G is a cut-vertex if and only if there exist vertices u and w $(\neq v)$ such that every path connecting u and w contains v.
19. Let $A = [a_{ij}]$ be the adjacency matrix of a graph G. Prove that (i, j)th entry $[A^n]_{ij}$ in A^n is the number of walks of length n from v_i to v_j .

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that for any graph *G*, $q(G) \geq p(G) - \omega(G)$.
21. Prove that A non trivial connected graph is Eulerian if and only if it has no vertex of odd degree.
22. Prove that a (p, q) – graph G is bipartite if and only if it contains no odd cycles.
23. (i) Prove if G is a plane (*p, q*) – graph in which every face is bounded by a cycle of length at least n, then $q \leq \dfrac{n(p-2)}{n-2}$

    (ii) Prove that if G is a planar (*p, q*) – graph $(p \geq 3)$ then $q \geq 3p - 6$.

    (iii) Prove that if G is a planar (*p, q*) – graph, then $\delta(G) \leq 5$.
24. If G is a (*p, q*) – graph , prove that $\chi(G) \geq \dfrac{p^2}{p^2 - 2q}$