B.Sc.DEGREE EXAMINATION,APRIL 2020
III Year VI Semester
Linear Algebra

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. If V is a vector space over F then prove that

   (i) $\alpha 0{=}0 \ for \ \alpha{\in}F$.

   (ii) $0v{=}0 \ for \ v \in V$
2. Define Quotient space.
3. If $dim_F V{=}m$ then, prove that $dim_F Hom\,(V, F)\,{=}m$.
4. Define annihilator.
5. Define inner product space of a vector space.
6. Define orthonormal set in vector space.
7. Define an algebra over a field.
8. Define an characteristic root.
9. Define an invariant under transformation.
10. When a linear transformation is said to be similar?
11. If $\{v_i\}$ is an orthonormal set, then prove that the vectors in $\{v_i\}$ are linearly independent.
12. Define dual space.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If $v_1, \ldots..v_n$ are in V then either they are linearly independent or some $v_k$is a linear combination of the preceding ones, $v_1, \ldots\ldots.v_{k-1}$.
14. If $v_1, \ldots..v_n$ is a basis of $V over F$ and if $w_1, \ldots\ldots.w_m$in V are linearly independent over F , then prove that $m \leq n$.
15. State and prove Schwarz inquality.
16. If $\lambda{\in}F$ is a characteristic root of $\in A(V)$, then prove that for any polynomial $q(x) \in F[x]$,$q(\lambda)$ is a characteristic root of $q(T)$.

17. If V is n-dimensional over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis $v_1, ..., v_n$ and the matrix $m_2(T)$ in the basis $w_1, ..., w_n$ of Vover$F$,then prove that there is an element $C \in F_n$ such that $m_2(T) = C\ m_1(T)\ \ C^{-1}$.
18. If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over$F$.
19. If V and W are of dimensions m and n , respectively, over F, then prove that $Hom(V, W)$ is of dimension mn over F .

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. (a) Prove that $L(S)$ is a subspace of V.

    (b) If V is the internal direct sum of $U_1, \ldots\ldots U_n$ then prove that V is isomorphic to the external direct sum of $U_1, \ldots\ldots U_n$.
21. If V is finite-dimensional and if W is a subspace of V, then prove that W is finite-dimensional, $dim\ W = dim\ V\ and\ dim\ V/W = dim\ V - dim\ W$.
22. Let V be a finite – dimensional inner product space; then prove that V has an orthonormal set as a basis.
23. If V is finite - dimensional over F then prove that for $S, T \in A(V)$
    a. $\gamma(ST) \leq \gamma(T)$;
    b. $\gamma(TS)) \leq \gamma(T)$
    $(and\ so, \gamma(ST) \leq min\ \gamma(T), \gamma(S))$
    c. $\gamma(ST) = \gamma(TS) = \gamma(T)\ for\ S\ regular\ in\ A(V)$.
24. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is basis of V in which the matrix of T is triangular.