B.Sc.DEGREE EXAMINATION,APRIL 2020
III Year VI Semester
Complex Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Sketch and state whether $|\mathbf{2z+3}| > \mathbf{4}$ is domain or not.
2. Show that the function $\mathbf{f(z)=\overline{z}}$ is nowhere differentiable.
3. Evaluate $\int_C \frac{\mathbf{z+2}}{\mathbf{z}}\mathbf{d}z$, where C is the semi circle $\mathbf{z=2e^{i\theta}}$, $(\mathbf{0} \leq \theta \leq \mathbf{2}\pi)$.
4. State Cauchy-Goursat theorem.
5. Evaluate $\int_C \frac{\mathbf{z}}{\mathbf{2z+1}}\mathbf{d}z$, where C denote the boundary of the square whose sides lie along the line $\mathbf{x=\pm 2}$ and $\mathbf{y=\pm 2}$ described in the positive sense.
6. State Maclaurin's series.
7. Using theorem involving a single residue evaluate $\int_C \frac{\mathbf{1}}{\mathbf{z}}\mathbf{d}z$, where C is the circle $|z|=2$ described in the positive sense.
8. Find the singularities of $\frac{\mathbf{s}inz}{\mathbf{z}}$ and mention the type of singularity.
9. Show that the transformation $\mathbf{w=i}z\mathbf{+i}$ maps the half plane $\mathbf{x>0}$ onto the half plane $\mathbf{v>1}$.
10. Find the fixed points of the transformation $\mathbf{w=}\frac{\mathbf{z-1}}{\mathbf{z+1}}$
11. Define singular point of a function.
12. Define essential singularity.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Find the Cauchy-Riemann equations in polar form.
14. If $\mathbf{f(z)=}\pi\mathbf{e}xp(\pi\mathbf{\overline{z}})$ and C is the boundary of the square with vertices at the points $\mathbf{0}$, $\mathbf{1}$, $\mathbf{1+i}$ $\mathbf{a}nd$ $\mathbf{i}$, the orientation of C being in the counter clock wise direction.
15. State and prove fundamental theorem of algebra.

16. Show that any singular point of $\mathbf{f}\left(\mathbf{z}\right)=\left(\frac{\mathbf{z}}{\mathbf{z+1}}\right)^{\mathbf{3}}$ is a pole. Further, determine the order m of pole and find the corresponding residue.
17. Find the transformation that maps the points $\mathbf{2}$, $\mathbf{i}$, $-\mathbf{2}$ in the z plane maps into the points $\mathbf{1}$, $\mathbf{i}$, $-\mathbf{1}$ in w-plane.
18. If $\mathbf{f}\left(\mathbf{z}\right)=\mathbf{u}\left(\mathbf{x},\mathbf{y}\right)+\mathbf{i}(\mathbf{x},\mathbf{y})$ is an analytic function and $\mathbf{u}\left(\mathbf{x},\mathbf{y}\right)=\frac{\mathbf{s}in\mathbf{2x}}{\mathbf{c}osh\mathbf{2y}-\mathbf{c}os\mathbf{2x}}$ find $\mathbf{f}\left(\mathbf{z}\right)$.
19. State and prove Cauchy Residue theorem.

Section C $(3\times 10=30)$ Marks

Answer any **THREE** questions

20. State and prove the sufficient condition for a function $\mathbf{f}(\mathbf{z})$ to have derivative at a point $\mathbf{z_0}$.
21. State and prove Cauchy integral formula.
22. State and prove Laurent's theorem.
23. Prove that and isolated singular point $\mathbf{z_0}$ of a function f is a pole of order m if and only if $\mathbf{f}\left(\mathbf{z}\right)$ can be written in the form $\mathbf{f}\left(\mathbf{z}\right)=\frac{\emptyset(\mathbf{z})}{(\mathbf{z}-\mathbf{z_0})^{\mathbf{m}}}$ where $\emptyset(\mathbf{z})$ is analytical and non zero at $\mathbf{z_0}$. Also, prove $\mathbf{R}es_{\mathbf{z}=\mathbf{z_0}}\,\mathbf{f}\left(\mathbf{z}\right)=\begin{cases}\emptyset\left(\mathbf{z_0}\right) & \mathbf{i}f\ \mathbf{m=1}\\ \frac{\emptyset^{(\mathbf{m-1})(\mathbf{z_0})}}{(\mathbf{m-1})!} & \mathbf{i}f\ \mathbf{m}\geq\mathbf{2}\end{cases}$.
24. Discuss the transformation $\mathbf{w}=\mathbf{s}inz$.

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8. Find the singularities of $\frac{sinz}{z}$ and mention the type of singularity.
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