M.Sc. DEGREE EXAMINATION,APRIL 2020
II Year IV Semester
Calculus of Variations and Integral Equations

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Linear functional.
2. Define a variation.
3. Define Weierstrass function.
4. State Legendre Condition.
5. Define Convolution Integral.
6. State Fredholm Alternative Theorem.
7. Define Neumann series.
8. State Fredholm's Third Theorem.
9. Define Complex Hilbert space.
10. Define Cauchy-Type Integral equation.
11. Define Degenerate Kernel.
12. State Mercer's Theorem.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Find a curve with specified boundary points whose rotation about the axis of abscissas generates a surface of minimum area.
14. Derive Jacobi's equation
15. Invert the integral equation $g(s) = f(s) + \lambda \int_0^{2\pi} (Sins\ Cost) g(t) dt$
16. Solve the Volterra equation $g(s) = 1 + \int_0^s stg(t)dt$
17. Solve the integral equation
$$f(s) = \int_a^s \frac{g(t)dt}{(cost - coss)^{\frac{1}{z}}},\ 0 \le a \le s \le b \le \pi$$
18. Test for an extremum the functional $v[y(x)] = \displaystyle\int_0^a y'^3 dx, y(0) = 0,$
$y(a) = b, a \ge 0, b \ge 0$

19. Solve the symmetric integral equation
$g(s) = (s+1)^2 + \int_{-1}^{1}(st + s^2t^2)g(t)dt$

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Derive Euler's equation.
21. Derive Transversality condition.
22. Show that the integral equation $g(s) = f(s) + (\frac{1}{\pi})\int_0^{2\pi} sin(s+t)g(t)dt$ possess no solution for f(s) = s but that it possesses infinitely many solution when f(s) = 1.
23. Solve the Fredholm integral equation $g(s) = 1 + \lambda\int_0^1(1-3st)g(t)dt$ and evaluate the resolvent kernel.
24. State and prove Abel Integral equation.