# M.Sc. DEGREE EXAMINATION,APRIL 2020
## I Year I Semester
## Algebra I

**Time : 3 Hours** **Max.marks :75**

**Section A** $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. If $a, b \in G$, When is b said to be conjugate of a?
2. Define p-Sylow subgroup.
3. Define invariants of a group.
4. Define solvable group.
5. Define trace of a matrix and find the trace of $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$.
6. If $T \in A(V)$ is Hermitian, Prove that all its characteristic roots are real.
7. Define finite field and give an example.
8. Define division ring.
9. Define norm of x in the division ring of real quaternions.
10. With usual notation, Show that $(xy)^* = y^* x^*$.
11. Define normalizer of a group.
12. With usual notation,for all $a, b \in F_n$, Show that $(AB)' = B'A'$.

**Section B** $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If G is a finite group, Prove that $c_a = \sum \frac{o(G)}{o(N(a))}$.
14. Prove that $S_{pk}$ has a p-Sylow subgroup.
15. Prove that G is solvable if and only if $G^{(k)} =$ (e) for some integer k.
16. Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.
17. State and prove the four square theorem.
18. Prove that for every prime number *p* and every positive interger *m* there exists a field having $p^m$ elements.

19. If $p(x) \in F[x]$is solvable by radicals over F, Prove that the Galois group over F of p(x) is a solvable group.

**Section C** $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. State and prove Sylow's theorem.
21. Prove that two abelian groups of order $p^n$ are isomorphic if and only if they have the same invariants.
22. If F is a field of characteristic 0 and if $T \in A(V)$ is such that $T^i = 0$ for all $i \geq 1$ Prove that *T* is nilpotent.
23. State and prove Wedderburn's theorem.
24. State and prove Frobenius theorem.