M.Sc. DEGREE EXAMINATION,APRIL 2020
I Year II Semester
Algebra II

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. When do you say that an element $a \in K$ is algebraic of degree n over F.
2. Define algebraic and transcendental numbers.
3. Write the remainder theorem.
4. For any $f(x), g(x)$ in $F[x]$, Show that $(f(x) + g(x))' = f'(x) + g'(x)$
5. Define fixed field of a group.
6. Define normal extension of a field.
7. Define field automorphism and when do you say that two automorphisms of a field are distinct.
8. Define $G(K, F)$.
9. Define invariant subspace of a vector space.
10. Define index of nilpotence of a linear transformation.
11. Write down the Jordan form of a linear transformation.
12. Define companion matrix of a polynomial.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If L is an algebraic extension of K and if K is an algebraic extension of F, Prove that L is an algebraic extension of F. Also prove that $[L : F] = [L : K][K : F]$
14. Show that $\sqrt{2} + \sqrt[3]{5}$ is algebraic over Q of degree 6.
15. Prove that a polynomial of degree n over a field can have at most n roots in any extension field.
16. Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f^{-1}(x)$ have a nontrivial common factor.
17. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.
18. If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, Prove that T satisfies a polynomial of degree n over F.

19. If $T \in A(V)$ has all its distinct characteristic roots $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n$ in F, Prove that a basis of V can be found in which the matrix T is of the form $\begin{pmatrix} J_1 & & & \\ & J_2 & & \\ & & ... & \\ & & & J_K \end{pmatrix}$. where each, $\begin{pmatrix} B_{i1} & & & \\ & B_{i2} & & \\ & & ... & \\ & & & B_{ir_i} \end{pmatrix}$, $B_{i1}, B_{i2}, ..., B_{ir_i}$ are basic Jordan blocks belonging to λ_i

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that the number e is transcendental.
21. If F is of characteristic 0 and if a, b are algebraic over F, Prove that there exists an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.
22. If K is a finite extension of F, Prove that $G(K, F)$ is a finite group and its order $o(G(K, F)$ satisfies $o(G(K, F)) \leq [k : F]$
23. If $T \in A(V)$ has all its characteristic roots in F, Prove that there is a basis of V in which the matrix of T is triangular.
24. Prove that the minimal polynomial of T_i is $q_i(x)^{i_i}$ for each $i = 1, 2, \ldots, k$,$V_i \neq (0)$ and $V = V_1 \oplus V_2 \oplus ... \oplus V_k$

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