M.Sc. DEGREE EXAMINATION,APRIL 2020
II Year III Semester
Differential Equations

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. State the second order linear homogeneous equation.
2. Define analytic function.
3. State Legendre equation of order p.
4. Write down the first order non-homogeneous linear equation.
5. What is solution matrix?.
6. Determine whether the function f(t,x) $=x^{1/2}$ be defined on the rectangle R $= \{$(t,x) : $|t| \leq 2$, $|x| \leq 2\}$ satisfy the Lipschitz condition.
7. State Picard's theorem.
8. Define order of a partial differential equation.
9. Write down the general form of first order linear partial differential equation.
10. Write the Charpit's auxillary equation for the partial differential equation $(p^2+q^2)y=qz$.
11. When will you say that the operator F(D,D') is reducible?
12. State the condition that the equation Rr +Ss+Tt+f(x,y,z,p,q)=0 to be elliptic.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Consider the differential equation x'=g(t), x(0)=0 where $g(t) = \begin{cases} \exp\left(-t^{-4}\right) & if\ t \neq 0 \\ 0 & if\ t = 0 \end{cases}$. Prove that the power series solution fails to exist.
14. Find e^{At} if A $= \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.
15. Calculate the first three successive approximation for the solution of the equation x'=tx, x(0) = 1.
16. Eliminate the arbitrary function f from the relation $z=xy+f(x^2+y^2)$.
17. Show that the equations xp-yq=x, $x^2p+q=xz$ are compatible.

18. If u = f(x+iy) + g(x-iy) , where f and g are arbitrary functions, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

19. Show that the equation $\frac{\partial^2 y}{\partial t^2} + 2k\frac{\partial y}{\partial t} = c^2\frac{\partial^2 y}{\partial x^2}$ possesses solutions of the form $\sum_{r=0}^{\infty} C_r e^{-kt} cos\left(\alpha_r x + \in_r\right) cos\left(w_r t + \delta_r\right)$ where C_r , α_r, $\in_r$, δ_r are constants and $w_r^2 = \alpha_r^2$c^2-k^2.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If P$_n$ is a Legendre polynomial, then prove that $\int_{-1}^{1} P_n^2\left(t\right) dt = \frac{2}{2n+1}$.

21. Prove that the matrix $X\left(t\right) = \begin{pmatrix} e^{-3t} & te^{-3t} & e^{-3t}t^2/2! \\ 0 & e^{-3t} & te^{-3t} \\ 0 & 0 & e^{-3t} \end{pmatrix}$ is fundamental for some linear system of the form x'=A(t)x where x= $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and A = $\begin{pmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{pmatrix}$.

22. Assume that f(t) and g(t) are non-negative continuous functions for $t \geq$t$_0$. Let k >0 be a constant. Prove that the inequality f(t)$\leq$ k + $\int_{t_0}^{t} g\left(s\right)\ f\left(s\right) ds$, t$\geq$t$_0$ implies the inequality f(t)$\leq$ $kexp(\int_{t_0}^{t} g\left(s\right)\ ds), t \geq t_0$.

23. Use Charpit's method, solve the partial differential equation z^2=pqxy.

24. Reduce the equation $(n-1)^2\ \frac{\partial^2 z}{\partial x^2}$ - $y^{2n}\frac{\partial^2 z}{\partial y^2} = ny^{2n-1}\frac{\partial z}{\partial y}$ to canonical form and find its general solution.

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