

**M.Sc. DEGREE EXAMINATION, APRIL 2020**  
**II Year IV Semester**  
**Differential Geometry and Tensor Calculus**

**Time : 3 Hours**

**Max.marks :75**

**Section A** ( $10 \times 2 = 20$ ) Marks

Answer any **TEN** questions

1. Define an arc length.
2. Define an involute.
3. Define metric.
4. Calculate the fundamental coefficient's E,F,G and H for the paraboloid  
 $\vec{r} = (u, v, u^2, -v^2)$
5. Define Geodesic .
6. Define a helix.
7. Define a symmetric tensor.
8. Define tensor.
9. State the transformation for a covariant tensor of rank two.
10. Define the christoffel 3 index symbol of the first kind.
11. Define an osculating plane .
12. Define an osculating circle

**Section B** ( $5 \times 5 = 25$ ) Marks

Answer any **FIVE** questions

13. Calculate the curvature and torsion of the cubic curve given by  $\vec{r} = (u, u^2, u^3)$ .
14. On the paraboloid  $x^2 - y^2 = z$ . Find the orthogonal trajectories of the section by the planes  $z = \text{constant}$ .
15. Show that the curves  $u+v = \text{constant}$  are geodesic on a surface with  
 $metric(1 + u^2)du^2 - 2uvdudv + (1 + v^2)dv^2$
16. Prove that the sum of two tensor which have the same no of covariant or contra variant indices is again a tensor of the same type and rank as the given tensor.
17. Show that if  $g_{ij} = 0$  for all  $i \neq j$  Then  $\{i,j\} = 1$
18. Find the involute of the circular helix  $\vec{r} = (a \cos u, a \sin u, bu)$

19. compute the first fundamental magnitude for the surface

$$\vec{r} = (u \cos v, u \sin v, f(u))$$

**Section C** ( $3 \times 10 = 30$ ) Marks

Answer any **THREE** questions

20. State and prove Fundamental existence theorem for space curves.
21. Prove that if  $\theta$  is the angle at the point  $(u, v)$  between the two directions given by  $Pdu^2 + 2Qdudv + Rdv^2 = 0$  then  $\tan \theta = \frac{2H\sqrt{Q^2 - PR}}{ER - 2F\theta + GP}$ .
22. State and prove Gauss Bonnett Theorem.
23. Prove that  $\frac{\partial}{\partial x^i} \log \sqrt{g} = \begin{Bmatrix} \alpha \\ i\alpha \end{Bmatrix}$ .
24. State and prove Ricci's theorem.

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