M.Sc. DEGREE EXAMINATION,APRIL 2020
II Year IV Semester
Functional Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define normed linear space.
2. State Holder's inequality.
3. Define isometric isomorphism of the normed linear space N into N'.
4. Define Hilbert spaces.
5. Define Orthonormal sets.
6. Prove that $\|T^*\| = \|T\|$.
7. Define normal operator.
8. Define Banach algebra.
9. Define Topological divisors of zero.
10. Define Spectrum of an element x in Banach algebra A.
11. Define Compact Hausdorff space in the weak* topology.
12. Define Banach *-algebra.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If N is a normal linear space and x_0 is a non zero vector in N, then prove that these exist a functional $f_0 \in N^*$ such that to $(x_0) = \|x_0\|\, and\, \|f_0\| = 1$ Hahn-Banach Theorm.
14. State and prove Minkowski's inequality.
15. Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.
16. If M is a closed linear subspace of a Hilbert space H, then prove that $H = M \oplus M^{\perp}$.
17. If N is a normal operator on H, then prove that $\left\|N^2\right\| = \|N\|^2$.
18. Prove that $\sigma(x)$ is non-empty.
19. If x is a normal element in a B*-algebra,then prove that $\left\|x^2\right\| = \|x\|^2$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Let N and N' be normed linear spaces and T a linear transformation of N into N' then prove that the following conditions are all equivalent to one another:

    (i)T is continuous;

    (ii)T is continuous at the origin,in the sense that $x_n \to 0 \Rightarrow T(x_n) \to 0$;

    (iii)There exists a real number $K \geq 0$ with the property $\|T(x)\| \leq K \|x\|$ for every $x \in N$

    (iv)If S $= \{x : \|x\| \leq 1\}$ is the closed unit sphere in N then its image T(S) is a bounded set in N'.

21. State and prove the Open Mapping Theorem.
22. Let H be a Hilbert space and let f be an arbitrary functional in H*,then prove that there exists a unique vector y in H such that $f(x) = \langle x, y \rangle$, for every x in H.
23. Prove that the mapping $x \to x^{-1}$ of G into G is continuous and is therefore a homeomorphism of G onto itself.
24. If f_1 and f_2 are multiplicative functionals on A with the same null space M then prove that $f_1 = f_2$.

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