M.Sc. DEGREE EXAMINATION,APRIL 2020
I Year I Semester
Real Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Lebesgue outer Measure.
2. Show that the constant functions are measurable.
3. Show that f is nonempty measurable function then f = 0 a.e iff $\int f dx = 0$.
4. State Fatou's lemma.
5. Define uniform convergence of sequence of function $\{f_n\}$.
6. When do you say that $\{f_n\}$ is uniformly bounded?.
7. Define Linear transformation,
8. Define contraction mapping.
9. Define Fourier series.
10. State Weierstrass theorem.
11. State implicit theorem.
12. State Lebesgue's monotone convergene theorem.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Let c be any real number and let f & g be real valued measurable functions defined on the same measurable set E, then Show that f+c , cf , f+g are also Measurable.
14. Show that $\int_1^\infty dx/x = \infty$
15. State and Prove Cauchy criterion for uniform convergence.
16. Show that a linear operator A on a finite - dimensional vector space X is one-to-one iff the range of A is all of X.
17. State and Prove Bessel's inequality.
18. If X is a complete metric space ,and if Φ is a contraction of X into X then Show that there exists one and only one $x \in X$ such that $\Phi(x) = x$.
19. Show that there exists an uncountable sets of zero measure.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that the class $\acute{M}$ is σ - algebra
21. State and Prove Lebesgue's dominated convergence theorem.
22. Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that $lim f_n(t) = A_n$ as $t \to x(n = 1, 2, 3, , ,)$ then Show that A_n converges and $Lim f(t)$ as $t \to x$ is equal to lim A_n as $n \to \infty$.
23. State and prove Inverse function theorem.
24. State and Prove Taylor's theorem.

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