M.Sc. DEGREE EXAMINATION,APRIL 2020
Topology
I Year II Semester

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define complete metric space.
2. Find all topologies defined on the set X=$\{a, b\}$
3. Define open base and open subbase for a topology.
4. Define product topology.
5. When you say that a metric space has Bolzano-Weierstrass property?
6. State Tychonoff's theorem.
7. Prove that a topological space is a T_1 space if and only if each point is a closed set.
8. Define completely regular space.
9. Prove that any continuous image of a connected space is connected.
10. Define locally connected space.
11. State Urysohn's lemma
12. State cauchy's inequality.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Let X be a complete metric space, and let Y be a subspace of X. Then prove that Y is complete if and only if it is closed.
14. Prove that any closed subspace of a compact space is compact.
15. Show that a metric space is compact if and only if it is complete and totally bounded.
16. Prove that every compact Hausdorff space is normal.
17. Show that the spaces R^n and C^n are connected.
18. Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous if and only if f^{-1} (G) is open in whenever G is open in Y.
19. Prove that every compact subspace of a Hausdorff space is closed.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Let X be a topological space and A be a subset of X. Then prove that (i) $\overline{A} =$ A$\cup$D(A) (ii) A is closed if and only if A $\supseteq$D(A).
21. State and prove Heine-Borel theorem.
22. Prove that in a sequentially compact metric space, every open cover has a lebesgue number.
23. State and prove Tietze extension theorem.
24. Let X be a compact Hausdorff space. Then prove that X is totally disconnected if and only it has an open base whose sets are also closed.

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