17UMACT4A07 UMA/CT/4A07

B.Sc. Degree Examinations - Even Semester 2021

II Year IV Semester

Vector Calculus and Fourier Transforms

Max Marks: 25

Answer any Five questions (5 * 5 = 25)

1. Find the directional derivative of $xyz - xy^2z^3$ at the point $(1,2,-1)$ in the direction of the vector $\vec{\imath} - \vec{\jmath} - 3\vec{k}$.

2. If $\vec{F} = 3xy\vec{\imath} - y^2\vec{\jmath}$, evaluate $\int_C \vec{F}.d\vec{r}$ where C is the curve on the $xy$ plane $y = 2x^2$ from (0,0) to (1,2).

3. Evaluate $\iiint_V \nabla.\vec{F}dV$ if $\vec{F} = x^2\vec{\imath} + y^2\vec{\jmath} + z^2\vec{k}$ and if V is the volume of the region enclosed by the cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$.

4. Find Fourier cosine transform of $f(x) = \begin{cases} x\,, 0 < x < 1 \\ 2 - x\,, \ 1 < x < 2 \\ 0\,, \ x > 2 \end{cases}$

5. Determine Inverse Fourier transform of $\frac{1}{12-7i\lambda-\lambda^2}$ by using Convolution theorem.

6. Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{\imath} + (3xz + 2xy)\vec{\jmath} + (3xy - 2xz + 2z)\vec{k}$ is irrotational and solenoidal.

7. Find the work done in moving a particle once around a circle C in the $xy$ plane if the circle has centre at the origin and radius 2 units and if the force field is given by
$\vec{F} = (2x - y + 2z)\vec{\imath} + (x + y - z^2)\vec{\jmath} + (3x - 2y - 5z)\vec{k}$.