

**M.PHIL. (STATISTICS) DEGREE EXAMINATIONS, EVEN SEMESTER 2021**

**I YEAR I SEMESTER**

**Research Methodology**

**Maximum Marks : 75**

**SECTION – A (5 X 15 = 75 marks)**

**(Answer any FIVE questions)**

1. (a) Write a note on definite quadratic form  $X'AX$ . If  $X'AX$  is positive definite, Then prove that (i)  $\det A > 0$  and (ii) every principal minor of  $A$  is positive

(b) Given the random variables  $X, Y, Z$  satisfying

$$X = 3Y + 2Z, \text{Var}[Y] = \text{Var}[Z] = 1, E[Y] = 2, \text{Cov}[Y, Z] = -0.5, E[Z] = -3$$

**Compute the following:**

- (i)  $E[X]$ .
- (ii)  $\text{Var}[X]$ .
- (iii)  $\text{Cov}[X, Y]$

2. (a) Define generalized inverse. Prove that a generalized inverse always exists although it is not unique.

(b) For any matrix  $A: m \times n$  and any g-inverse  $A^- : m \times n$ , then

- (i)  $AA^-$  and  $A^-A$  are idempotent
- (ii)  $\text{rank}(A) = \text{rank}(AA^-) = \text{rank}(A^-A)$
- (iii)  $\text{rank}(A) \leq \text{rank}(A^-)$

3. (a) State and prove Monotone convergence theorem.

(b) Define convergence in probability and convergence in distribution. Also prove that the convergence in probability implies convergence in distribution.

4. (a) State and prove Central Limit theorem.  
(b) Discuss Kolmogorov strong law of large numbers.
5. Explain in detail how the genetic algorithm works?
6. (a) What do you mean by bivariate distribution? Derive PGF of bivariate Poisson distribution.  
(b) Explain in detail bootstrapping technique with example.
7. (a) Explain in detail the criteria of a good research.  
(b) Discuss the layout of a research report.
8. (a) Define research. List out its types.  
(b) Explain the types of optimization techniques.