M.PHIL. (STATISTICS) DEGREE EXAMINATIONS, EVEN SEMESTER 2021

I YEAR I SEMESTER

Advanced Statistical Inference

Maximum Marks : 75

SECTION – A (5 X 15 = 75 marks)

(Answer any FIVE questions)

1. **(a) Stat and prove Neyman Fisher Facroisation theorem.**

   (b) Let X_1, X_2, …,X_n be a random sample of size n from U[0, θ], $\theta > 0$. Find the Sufficient Statistic for θ .

2. **(a) State and establish Rao-Blackwell theorem**

   (b) Explain completeness and boundedly completeness with an illustration. (7)

3. **(a) State the generalized Neyman - Pearson lemma.**

   (b) Let X_1, X_2, …,X_n be a random sample from N[μ, σ^2], $\theta \in$ R. Derive UMP level α test for testing the hypothesis $H_0 : \mu \leq \mu_0$ against $H_1 : \mu > \mu_0$ (8)

4. **(a) Define multi Parameter exponential family. Also mention its objectives and properties.**

   (b) Let X_1, X_2, …,X_m be a random sample from P(λ) and Y_1, Y_2, …,Y_n be a random sample from P(μ) . Derive UMPU level α test for testing the hypothesis H : $\lambda \leq \mu$ Vs K: $\lambda > \mu$.

5. **(a) What is linear rank statistic? Establish its variance.**

   (b) Explain Kruskal - Wallis test in more than two sample problem.

6. (a) State and Prove a necessary and sufficient condition for an estimator to be UMVUE using uncorrelatedness approach.

(b) Explain Friedman's two ways analysis of variance by ranks test in detail.

7. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N[\mu, \sigma^2]$, $\mu \in R$, $\sigma^2 > 0$ Show that $\bar{X}$ is sufficient statistic for μ when σ^2 known.

(b) Let X have the distribution $P \in \mathcal{P}$ and T be a sufficient statistic for $\mathcal{P}$. Show that a necessary and sufficient condition for all similar tests have Neyman structure is that the family $\mathcal{P}^T$ of distributions of T is boundedly complete

8. (a) State and establish Lehmann-Scheffe theorem

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N[\mu, \sigma^2]$. Obtain the minimal sufficient statistic.