

M.Sc. DEGREE EXAMINATION, ODD SEMESTER 2020
II Year III Semester
DIFFERENTIAL EQUATIONS

Max.marks :25

Answer any **FIVE** questions ($5 \times 5 = 25$) Marks

1. Consider the differential equation $x' = g(t)$, $x(0) = 0$ where

$$g(t) = \begin{cases} \exp(-t^{-4}) & \text{if } t \neq 0 \\ 0 & \text{if } t = 0 \end{cases}$$
 Prove that the power series solution fails to exist.
2. Find e^{At} if $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.
3. Calculate the first three successive approximation for the solution of the equation $x' = tx$, $x(0) = 1$.
4. Eliminate the arbitrary function f from the relation $z = xy + f(x^2 + y^2)$.
5. Show that the equations $xp - yq = x$, $x^2p + q = xz$ are compatible.
6. If $u = f(x + iy) + g(x - iy)$, where f and g are arbitrary functions, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$
7. Show that the equation $\frac{\partial^2 y}{\partial t^2} + 2k \frac{\partial y}{\partial t} = c^2 \frac{\partial^2 y}{\partial x^2}$ possesses solutions of the form

$$\sum_{r=0}^{\infty} C_r e^{-kt} \cos(\alpha_r x + \epsilon_r) \cos(w_r t + \delta_r)$$
 where $C_r, \alpha_r, \epsilon_r, \delta_r$ are constants and

$$w_r^2 = \alpha_r^2 c^2 - k^2.$$