

B.Sc. DEGREE EXAMINATION, ODD SEMESTER 2020
III Year V Semester
GRAPH THEORY

Max.marks :25

Answer any **FIVE** questions ($5 \times 5 = 25$) Marks

1. Define induced sub graph, edge induced sub graph and spanning sub graph.
2. If G is a Hamiltonian graph, prove that $\omega(G-S) \leq |S|$, for every non-empty subset S of $V(G)$.
3. Prove that a graph G is a tree if and only if every two vertices of G are connected by a unique path.
4. Prove that A graph is planar if and only if it contains no contraction of K_5 or $K_{3,3}$.
5. Prove that there exists a k -colouring of a graph G if and only if $V(G)$ can be partitioned into k subsets V_1, V_2, \dots, V_k such that no two vertices in V_i , $i=1, 2, \dots, k$ are adjacent.
6. Prove that a vertex v in a connected graph G is a cut-vertex if and only if there exist vertices u and w ($\neq v$) such that every path connecting u and w contains v .
7. Let $A = [a_{ij}]$ be the adjacency matrix of a graph G . Prove that (i, j) th entry $[A^n]_{ij}$ in A^n is the number of walks of length n from v_i to v_j .