SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC)

Chromepet,Chennai — 600 044.

B.Sc. END SEMESTER EXAMINATION APRIL/NOV – 2021

SEMESTER - V

08UMACT5012 **&** UMA/CT/5012 - **Graph Theory**

Total Duration	**: 3 Hrs**	**Total Marks**	**: 75**
MCQ	: 30 Mins	MCQ	: 15
Descriptive	: 2 Hrs.30 Mins	Descriptive	: 60

Section B

Answer any ***SIX*** questions (6 × 5 = 30 Marks)

1. Show that if $\delta(G) \geq \frac{p-1}{2}$, then G is connected.
2. If G is a (p, q) – graph with $p \geq 3$ and $\geq \frac{p^2-3p+6}{2}$, prove that G is Hamiltonian.
3. If G is connected graph, prove that the distance between v_i and v_j is the smallest integer n (≥ 0) such that $[A^n]_{ij} \neq 0$.
4. If G is a connected plane (p, q) – graph with r number of faces, prove that $p - q + r = 2$.
5. Prove that for any graph G, $\chi(G) \leq \Delta(G) + 1$.
6. Prove that every non trivial graph contains at least two vertices which are not cut vertices.
7. If G is a Hamiltonian graph, prove that $\omega(G - S) \leq |S|$, for every non empty subset S of V(G).
8. Let G ($\delta(G) > 0$) be a bipartite graph with bipartition [A, b], where $|A| = n$. If $\min_{x \in A} \deg(x) \geq \max_{y \in B} \deg(y)$, prove that G contains n independent edges.

Section C

Answer any ***THREE*** questions (3 × 10 = 30 Marks)

9. If $q > \frac{p^2}{4}$, prove that every $(p, q) - graph$ contains a triangle.
10. Prove that a nontrivial connected graph is eulerian if and only if it has no vertex of odd degree.
11. Prove that a $(p, q) - graph\ G$ is bipartite graph if and only if it contains no odd cycles.
12. Prove that there are exactly five regular polyhedra.
13. Prove that for any given integer $k(\geq 1)$ there exists a triangle free graph with chromatic number k.