17UMACT5A10 & UMA/CT/5A10 - **Real Analysis**

Total Duration : 3 Hrs	**Total Marks : 75**
MCQ : 30 Mins	MCQ : 15
Descriptive : 2 Hrs.30 Mins	Descriptive : 60

Section B

Answer any *SIX* questions (6 × 5 = 30 Marks)

1. Prove that the set of rational numbers is countable.

2. If $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of real numbers, then show that

$$\lim_{n \to \infty} \sup s_n = \lim_{n \to \infty} s_n .$$

3. Prove that $\lim_{x \to 1} \sqrt{x+3} = 2$.

4. Show that a totally bounded subset of a metric space is totally bounded.

5. State and prove Rolle's Theorem

6. Show that a nondecreasing sequence which is bounded above is convergent.

7. Prove that if $\displaystyle\sum_{n=1}^{\infty} a_n$ converges absolutely , then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

8. Show that if E is any subset of a metric space M, then $\overline{E}$ is closed.

Section C

Answer any *THREE* questions (3 × 10 = 30 Marks)

9. Prove that the set [0,1] is uncountable.

10. Show that any bounded sequence of real numbers has a convergent subsequence.

11. If $\langle M_1, \rho_1 \rangle$ and $\langle M_2, \rho_2 \rangle$ are metric spaces and $f : M_1 \to M_2$, then prove that f is continuous on M_1 if and only if f^{-1} is open in M_1 whenever G is open in M_2 .

12. If $\langle M, \rho \rangle$ is a complete metric space, T is a contraction on M, then show that there is one and only one point x in M such that $Tx = x$.

13. State and prove the Chain rule for derivatives.