SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC)
Chromepet,Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATION APRIL/NOV – 2021
SEMESTER - V
21UMACT5009 - Modern Algebra

Total Duration	**: 3 Hrs**	**Total Marks**	**: 75**
MCQ	: 30 Mins	MCQ	: 15
Descriptive	: 2 Hrs.30 Mins	Descriptive	: 60

Section B

Answer any ***SIX*** questions (6 × 5 = 30 Marks)

1. If G is a finite group and $a \in G$, then prove that $O(a) \,/\, O(G)$.

2. If G is group and H is a subgroup of index 2 in G, prove that H is a normal subgroup G.

3. If $\emptyset$ is a homomorphism of G into $\bar{G}$ with kernel K, prove that K is a normal subgroup of G.

4. If p is a prime number, prove that J_p, the ring of integers $mod\ p$, is a filed.

5. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Prove that R is a filed.

6. Let R be a Euclidean ring and $a, b \in R$. If $b \neq 0$ is not a unit in R, prove that $d(a) < d(ab)$.

7. Express as the product of disjoint cycles : $(1,2,3)(4,5)(1,6,7,8,9)(1,5)$

8. If U, V are ideals of R, let $U + V = \{u + v \mid u \in U, v \in V\}$. Prove that $U + V$ is also an ideal.

Contd…

Section C

Answer any ***THREE*** questions (3 × 10 = 30 Marks)

9. If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$, respectively, prove that $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$.

10. Let $\emptyset$ be a homomorphism of G onto $\bar{G}$ with kernel K, prove that $G/K \approx \bar{G}$.

11. Prove that a finite integral domain is a filed.

12. If U is an ideal of the ring R, prove that R/U is a ring and is a homomorphic image of R.

13. Prove that every integral domain can be imbedded in a field.