SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS APRIL-2022
SEMESTER - II
20USTAT2002 - Allied Mathematics - II

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If f:A$\rightarrow$B and if X $\subset$ B,Y $\subset$ B then show that f^{-1}(X$\cup$Y) = $f^{-1}(X) \cup f^{-1}(Y)$.
2. Define the following terms and give an example
   i. Convergent sequence.
   ii. Divergent sequence.
   iii. Monotone sequence.
3. If f and g both have derivatives at $c \in R^1$ then show that $f + g$ and fg are also differentiable at c.
4. If L$[f(t)] = F[s]$ then prove that the following
   i. L$[f(at)] = \frac{1}{a}$ F$\left[\frac{s}{a}\right]$, a>0.
   ii. L$[e^{at}\ f(t)] = F$[s-a].
5. Find the inverse Laplace transform for the following
   i. $\dfrac{s}{(s-4)^2}$
   ii. $\dfrac{s}{(s^2+2s+10)}$
6. Answer the following questions,
   i. If f,g are function from $\mathbb{R}$ to $\mathbb{R}$ given by $f(x)= 2x$, $g(x) = x^2$ then find f $\circ$ g and g $\circ$ f
   ii. If f:$\mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x)= x^2$ then find f^{-1}(9), f^{-1}(-9).
   iii. Define countable set and give an example.
7. State and prove Rolle's theorem.
8. Find the Laplace transform of the following
   i. $\left[e^t + \frac{1}{e^t}\right]^2$
   ii. $\cos^3$t
   iii. 3cosh2t.

Contd...

Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Prove that the countable union of countable sets is countable, and hence show that the set of all irrational numbers is countable.
10. Prove that the sequence $\{s_n\}_{n=1}^{\infty}$, where $s_n = \left(1 + \frac{1}{n}\right)^n$ is convergent.
11. State and prove Taylor's theorem with integral form of the remainder.
12. Evaluate the following
    i. $L[e^{-3t} \sin t \cos t]$
    ii. $L[t^2(\cosh 3t + \sinh 3t)]$
13. Evaluate : $L^{-1}\left[\frac{4s+5}{(s-1)^2+(s+2)}\right]$

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