SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. END SEMESTER EXAMINATION APRIL/NOV - 2021
SEMESTER – II

17PAMCE2002– Mathematical Statistics

Total Duration : 3 hrs		**Total Mark : 75**	
MCQ	: 30 min	MCQ	: 15
Descriptive	: 2 Hrs. 30 Mins.	Descriptive	: 60

Section B

Answer any ***Six*** questions (6 x 5 =30)

1. State and prove factorization criterion theorem.

2. Prove that if T is a complete sufficient statistic and there exists an unbiased estimator h of θ, which is given by E{h/T}.

3. Let T(X, θ) be a pivot such that for each θ, T(X, θ) is a static, and as a function of θ, T is either strictly increasing or decreasing at each $x \in R_n$. Let $\wedge \subseteq R$ be the range of T, and for every $\lambda \in \wedge$ and $x \in R_n$, let the equation $\lambda = T(X,\theta)$ be solvable. Then Prove that one can construct a confidence interval for θ at any level.

4. Let T(x) be maximal invariant with respect to ς. Then prove that φ is invariant under ς if and only if φ is a function of T.

5. Nine adults agreed to test the efficacy of a new diet program. Their weights were measured before and after the program and found to be as follows

	Participant								
Measures	1	2	3	4	5	6	7	8	9
Before	132	139	126	114	122	132	142	119	126
After	124	141	118	116	114	132	145	123	121

6. The lifetimes(in hours) of samples from three different brands of batteries, Y1,Y2 and Y3, were recorded, with the following results:

Contd…

Y_1	40	30	50	50	30	
Y_2	60	40	55	65		
Y_3	60	50	70	65	75	40

To test whether the three brands have different average lifetimes. Assume that the three samples come from normal populations with common (unknown) standard deviation σ.(T.V. $F_{2,12,0.05}$=3.89)

7. Estimate β_0 and β_1 using method of least square.

8. In a 72 hour period on a long holiday weekend, there was a total of 306 fatal automobile accidents. The data as follows

No. Of Fatal accidents per hour	0 or1	2	3	4	5	6	7	8 or more
No. Of Hours	4	10	15	12	12	6	6	7

To test the hypothesis that the number of accidents per hour is a Poisson Random Variable.

Section C

Part A

Answer any ***Two*** questions (2x 10 =20)

9. State and prove Cramer-Roa theorem.

10. Let $X_1, X_2, \dots X_n$ be sample from $N(\mu, \sigma^2)$, where σ^2 is known. Then $\overline{X}$ is sufficient for μ and take $T_\mu(X) = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}}$. then $1 - \alpha = P\left\{a < \dfrac{\overline{X} - \mu}{\sigma}\sqrt{n} < b\right\} = P\left\{\overline{X} - b\dfrac{\sigma}{\sqrt{n}} < \mu < \overline{X} - a\dfrac{\sigma}{\sqrt{n}}\right\}$. Prove that the length of this confidence interval is $(\sigma / \sqrt{n})(b - a)$.

11. State and prove Neyman-Pearson Fundamental Lemma.

12. Let $X \sim b(n, p)$ then find a level α likelihood ratio test of H_0: $p \le p_0$ against H_1: $p > p_0$: $\lambda(x) = \dfrac{\sup_{p \le p_0} \binom{n}{x} p^x (1-p)^{n-x}}{\sup_{0 \le p \le 1} \binom{n}{x} p^x (1-p)^{n-x}}$

Part B

Compulsory Question (1 x 10 = 10)

13. Explain Two-Way analysis of variance with one observation per cell.