SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. END SEMESTER EXAMINATION APRIL/NOV - 2021
SEMESTER - I
17PAMCT1A01- Algebra - I

Total Duration : 3 Hrs		**Total Marks : 75**	
MCQ	: 30 Mins	MCQ	: 15
Descriptive	: 2 Hrs.30 Mins	Descriptive	: 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks$)$

1. Prove that N(a) is a subgroup of G.

2. Prove that S_n is not solvable for n $\geq$ 5.

3. Prove that any finite abelian group is the direct product (sum) of cyclic groups.

4. If A is invertible then prove that tr (ACA^{-1}) = tr C.

5. If N is normal and AN = NA, then prove that AN* = N* A.

6. Let F be a finite field; Prove that F has p^m elements where the prime number p is the characteristic of F.

7. Prove that the general polynomial of degree n $\geq$ 5 is not solvable by radicals.

8. For all x,y $\in$ Q, prove that N(xy) = N(x)N(y).

Section C

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks$)$

9. Prove that G is solvable if and only if G $^{(k)}$= (e) for some integer k.

10. If (vT, vT) = (v, v) for all v $\in$ V, then prove that T is unitary.

11. If F is a finite field and $\alpha \neq \mathbf{o}$, $\beta \neq 0$ are two elements of F
    then we can find elements a and b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

12. Let C be the field of complex numbers and suppose that the division
    ring D is algebraic over C then prove that D = C.

Part B

Compulsory question $(1 \times 10 = 10$ Marks$)$

13. Prove that if a $\in$ H then $a^{-1} \in$ H if and only if N(a) = 1