SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. END SEMESTER EXAMINATION APRIL/NOV - 2021
SEMESTER – III
17PAMCT3A08 – DIFFERENTIAL EQUATIONS

Total Duration : 3 hrs	**Total Mark : 75**
MCQ : 30 min	MCQ : 15
Descriptive : 2 Hrs. 30 Mins.	Descriptive : 60

Section B

Answer any ***Six*** questions (6 x 5 =30)

1. The equation for the motion of a simple pendulum is $x''(t) + K\sin x(t) = 0$ where K is a constant , the equation with initial conditions $x(0) = \frac{\pi}{6}$ & $x'(0) = 0$ Find the series of solution.
2. If P_n is a Legendre polynomial then, Prove that $\int_{-1}^{1} P_n^2(t)dt = \frac{2}{2n+1}$
3. Consider the linear system $x' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{bmatrix} x$ find the fundamental matrix.
4. Consider the initial value problem $x' = x^2 + \cos^2 t, x(0)$ determine the largest interval of existence of its solution by using Picard's theorem.
5. State and prove Gronwall inequality.
6. Form the PDE by eliminating the arbitrary function from
   (i) $z = f(x+it) + g(x-it), where....i = \sqrt{-1}$
   (ii) $f(x+y+z, x^2+y^2+z^2) = 0.$
7. Find the complete integral of $x^2p^2 + y^2q^2 - 4 = 0$ using Charpit's method.
8. Solve the following PDE $(D^2 - 4DD' + 4D'^2)u = e^{2x+y}$.

Contd…

Section C

Part A

Answer any ***Two*** questions (2 x 10 =20)

9. Let $A_1, A_2, \ldots$ be the positive zeros of Bessels's function J_p then

   Prove that $\int_0^1 tJ_p(A_m t)J_p(A_n t)dt = \begin{cases} 0, \ldots\ldots\ldots\ldots\ldots if \ldots m \neq n; \\ \frac{1}{2} J_{p+1}(A_n)^2, if \ldots m = n \end{cases}$

10. State and prove Existence and uniqueness theorem.

11. State and prove Picard's theorem.

12. Find the general integral of the following linear partial differential equations

    (i) $y^2 p - xyq = x(z - 2y)$ (ii) $(y + zx)p - (x + yz)q = x^2 - y^2$

Part B

Compulsory Question (1 x 10 = 10)

13. Explain Canonical form for Parabolic Equation.