

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC) Chromepet,
Chennai — 600 044.

M.Sc. END SEMESTER EXAMINATION APRIL/NOV - 2021

SEMESTER - I

20PAMCT1001 - Algebra - I

Total Duration : 3 Hrs	Total Marks : 75
MCQ : 30 Mins	MCQ : 15
Descriptive : 2 Hrs.30 Mins	Descriptive : 60

Section B

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. If $O(G) = P^n$, where P is a prime number, then show that G has a non-trivial centre.
2. Let G be a group and suppose that G is the internal direct product of $N_1, N_2, N_3, \dots, N_r$. Let $T = N_1 \times N_2 \times N_3 \times \dots \times N_r$. Prove that G and T are isomorphic.
3. If $T \in A(V)$ then prove that $\text{tr } T$ is the sum of the characteristic roots of T .
4. Prove that for every prime number P and for every positive integer m , then there exist a field having P^m elements.
5. State and prove Lagrange's Identity.
6. Let G be a group of order 30. Show that G is not simple.
7. Show that kernel of homomorphism is a sub-module.
8. If λ is a characteristic root of the normal transformation N and if $Nv = \lambda v$ then prove that $N^*v = \bar{\lambda}v$.

Section C

Part A

Answer any **TWO** questions ($2 \times 10 = 20$ Marks)

9. State and prove the fundamental theorem on finitely generated modules over Euclidean rings.
10. State and prove Wedderburn's theorem.
11. Let C be the field of complex numbers and suppose that the division ring D is algebraic over C . Then prove that $D = C$.
12. If $T \in A(V)$, then prove that (a) $T^* \in A(V)$ (b) $(T^*)^* = T$ (c) $(S+T)^* = S^* + T^*$
(d) $(ST)^* = T^*S^*$

Part B

Compulsory question ($1 \times 10 = 10$ Marks)

13. State and prove First sylow theorem.