SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC) Chromepet, Chennai — 600 044.
M.Sc. END SEMESTER EXAMINATION APRIL/NOV - 2021
SEMESTER – III

20PAMCT3007– Complex Analysis

Total Duration : 3 hrs	**Total Mark : 75**
MCQ : 30 min	MCQ : 15
Descriptive : 2 Hrs. 30 Mins.	Descriptive : 60

Section B

Answer any ***Six*** questions (6 x 5 =30)

1. State and Prove Open Mapping Theorem.

2. Show that $\boldsymbol{for\ \ a > 1}, \int_0^{\pi} \frac{d\theta}{a+\cos\theta} = \frac{\pi}{\sqrt{a^2-1}}$.

3. If *f* is analytic in a region G and *a* is a point in G with $|f(a)| \geq |f(z)|$ for all z in G then prove that *f* must be a constant function.

4. Let Re z_n> -1. Prove that the series $\sum \boldsymbol{\log\,(1+z_n)}$ converges absolutely if the series$\sum \boldsymbol{z_n}$converges absolutely.

5. If *f* is a mesomorphic function on an open set G then prove that there are analytic functions g and h on G such that f = g/h

6. Let *G* be a region and $\boldsymbol{\varphi_1}$ and $\boldsymbol{\varphi_2}$ are subharmonic functions on G; If $\boldsymbol{\varphi(z) = \ \max\{\varphi_1(z), \varphi_2(z)\ \}}$ for each z in *G* then prove that $\boldsymbol{\varphi}$ is a Sub harmonic function.

7. Let G be a region in $\mathbb{C}$ and let S be a closed connected subset of $\mathbb{C}_\infty$ such that $\infty \in \mathbf{S}$ and S$\cap$ $\boldsymbol{\delta_\infty G = \{a\}}$. If G0 is the component of $\mathbb{C}_\infty - \mathbf{S}$ which contains G then prove that G0 is a simply connected region in the plane.

8. Suppose G is analytic on B(0;R), g(0) = 0, $\mathbf{|g'(0)| = \mu > 0}$ and $\mathbf{|g(z)| \leq M}$ for all z then prove that g(B(0;R))$\supset$ $\mathbf{B\left(0, \frac{R^2\mu^2}{6M}\right)}$.

Contd…

Section C

Part A

Answer any ***Two*** questions (2x 10 =20)

9. Let G be an open set and let f: G $\rightarrow \mathbb{C}$ be a differentiable function then prove that f is analytic on G.

10. Let f: D $\rightarrow$ D be a one-one analytic map on D onto itself and suppose f (a)=0 then prove that there is a complex number c with $|\mathbf{c}|$= 1 such that f = c$\boldsymbol{\varphi_{\alpha}}$

11. If G is a bounded Dirichlet Region then prove that for each a in G there is a Green's function on G with singularity at a.

12. For Re z > 1, then show that $\boldsymbol{\zeta(z)\Gamma(z)}=\int_{\mathbf{0}}^{\infty}(\boldsymbol{e^{t}}-\mathbf{1})^{-\mathbf{1}}\,\boldsymbol{t^{z-1}}dt$.

Part B

Compulsory Question (1 x 10 = 10)

13. For each $\boldsymbol{\alpha}$ **and** $\boldsymbol{\beta}$, 0<$\boldsymbol{\alpha} < \infty$ and , 0$\leq \boldsymbol{\beta} \leq \mathbf{1}$, there is a constant C($\boldsymbol{\alpha}, \boldsymbol{\beta}$) such that if f is an analytic function on some simply connected region containing $\overline{\mathbf{B}}$(0,1) that omits the values 0 and 1 , and such that $|\mathbf{f}(\mathbf{0})| \leq \boldsymbol{\alpha}$ then prove that $|\mathbf{f}(\mathbf{z})| \leq \mathbf{C}(\boldsymbol{\alpha}, \boldsymbol{\beta})$ for $|\mathbf{z}| \leq \boldsymbol{\beta}$.