**SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)**
**(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC)**
**Chromepet, Chennai — 600 044.**
**M.Sc. END SEMESTER EXAMINATION APRIL/NOV - 2021**
**SEMESTER – I**
**20PAMET1001 – Probability and Distributions**

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|---|---|---|---|
| **Total Duration : 3 hrs** | | **Total Mark** | **: 75** |
| MCQ | : 30 min | MCQ | : 15 |
| Descriptive | : 2 Hrs. 30 Mins. | Descriptive | : 60 |

## Section A

Answer any ***SIX*** questions (6 x 5 =30)

1. Define Negative binomial distribution. Derive mgf of Negative binomial distribution.
2. Define Hyper-Geometric distribution. Find the Mean and variance of this distribution.
3. The joint probability density function of a two-dimensional random variable (X,Y) is given by

$$f(x,y)=\begin{cases}2; & 0<x<1,\ 0<y<x;\\ 0, & elsewhere\end{cases}$$

(i) Find the marginal density functions of X and Y.

(ii) Find the conditional density function of Y given X=x and conditional density function of X given Y=y.

(iii) Check for independence of X and Y.

4. Prove that $(n-1)\frac{s^2}{\sigma^2}\ is\ \chi^2(n-1)$.
5. Prove that the sum of independent chi-square variates is also a $\chi^2$-variate.
6. Define

(a) Bivariate binomial distribution and write mean and variance of the distribution.

(b) Bivariate Normal distribution and write mean and variance of the distribution.

7. Let $\{X_n, Y_n\}$, n=1,2,...be a sequence of pairs of random variables and c be a constant. Then prove that $X_n \xrightarrow{L} X$, $Y_n \xrightarrow{P} c \Rightarrow X_n+Y_n \xrightarrow{L} X+c$.
8. Let $X_n \xrightarrow{P} X$ and g be a continuous function defined on R. Then prove that $g(X_n) \xrightarrow{P} g(X)$ as $n\to\infty$.

## Section B

### Part A

Answer any ***TWO*** questions (2 x 10 =20)

9. Let $X\sim C(\mu_1,\theta_1)$ and $Y\sim C(\mu_2,\theta_2)$ be independent random variable. Then prove that $X+Y$ is a $C(\mu_1+\mu_2,\theta_1+\theta_2)$ random variable.

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10. Let (X,Y) be jointly distributed with density function

$$f(x,y) = \begin{cases} x + y\,, 0 < x < 1, 0 < y < 1, \\ 0 \qquad\qquad otherwise \end{cases}$$. Find Mean,Variance and Covariance.

11. State and prove Lindeberg-Levy central limit theorem.

12. Define t-distribution. Derive the probability density function of t-distribution.

## Part B

Compulsory Question (1 x 10 = 10)

13. Derive the Mean, Variance and Moment generating function for the chi-square distribution.