SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC) Chromepet, Chennai — 600 044.

M.Sc. END SEMESTER EXAMINATION APRIL/NOV - 2021

SEMESTER – IV

08PAMCT4010 – Functional Analysis

Total Duration : 3 hrs		**Total Mark : 75**	
MCQ	: 30 min	MCQ	: 15
Descriptive	: 2 Hrs. 30 Mins.	Descriptive	: 60

Section A

Answer any ***SIX*** questions (6 x 5 =30)

1. State and prove Minkowski's inequality.
2. If N is a normed linear space and $x_0$is a non zero vector in N, then there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ *and* $\|f_0\| = 1$.
3. A closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.
4. If $\{e_i\}$ is an orthonormal set in a Hilbert space H, then $\sum|(xe_i)|^2 \leq \|x\|^2$ for every vector x in H.
5. If T is an operator on H for which $(Tx, x) = 0$ for all x, then $T = 0$.
6. The mapping $x \to x^{-1}$of G into G is continuous and is therefore a homeomorphism of G onto itself.
7. Prove that $\sigma(x)$ is non-empty
8. If f_1 and f_2 are multiplicative functionals on A with the same null space M, then$f_1 = f_2$.

Section B

Part A

Answer any ***TWO*** questions (2 x 10 =20)

9. Let H be a Hilbert Space and let f be an arbitrary functional in H^*. Then there exists a unique vector y in H such that $\mathrm{f(x) = (x, y)}$ for every x in H.
10. State and prove Uniform boundedness theorem.
11. Prove that $\mathrm{r(x) = lim\|x^n\|^{1/n}}$.
12. State and prove Gelfand-Neumark theorem.

Contd.…

Part B

Compulsory Question (1 x 10 = 10)

13. Let M be a closed linear subspace of a normed linear space N. If the norm of a coset $x + M$ in the quotient space N/M is defined by $\|x + M\| = inf\{\|x + m\|: m \in M\}$ then prove that N/M is a normed linear space. Further, if N is a Banach space, then prove that N/M is also a Banach space.