SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. END SEMESTER EXAMINATION APRIL/NOV - 2021
SEMESTER - I
14PAMCT1A02 - Real Analysis

Total Duration : 3 Hrs		**Total Marks : 75**	
MCQ	: 30 Mins	MCQ	: 15
Descriptive	: 2 Hrs.30 Mins	Descriptive	: 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Let f be a measurable function and B a Borel set. Prove that f^{-1} is a measurable set.

2. Show that there exist uncountable sets of zero measure.

3. Show that if f is a non-negative measurable function, then f= 0 a.e. if, and only if, f dx = 0.

4. Prove that $\mathbf{l}im \int_{0}^{\infty} \frac{\mathbf{d}x}{\left(\mathbf{1}+\frac{\mathbf{x}}{\mathbf{n}}\right)^{\mathbf{n}} \mathbf{x}^{\mathbf{1/n}}} = \mathbf{1}$.

5. Suppose K is compact, (a)$\{f_n\}$ is a sequence of continuous functions on K (b)$\{f_n\}$ converges pointwise to a continuous function f on K (c) $f_n(x) \geq$) f_{n+1} (x) for all x K, n = 1,2,3,...
   then prove that f_n to f uniformly on K.

6. Suppose f maps a convex open set E $\subset$ R^n into R^m, f is differentiable in E, and there is a real number M such that $\left\|\mathbf{f}^{'}(\mathbf{x})\right\| \leq \mathbf{M}\ \mathbf{f}or\ \mathbf{e}very\ \mathbf{x} \in \mathbf{E}$, then prove that $\left|\mathbf{f}\left(\mathbf{b}\right) - \mathbf{f}(\mathbf{a})\right| \leq \mathbf{M}\left|\mathbf{b} - \mathbf{a}\right|$ for all a$\in \mathbf{E}$, and b$\in \mathbf{E}$

7. Given a double sequence $\{a_{ij}\}$, i = 1,2,3,.., j=1,2,3,.... Suppose that $\sum_{\mathbf{j=1}}^{\infty} |\mathbf{a_{i}}_j| = \mathbf{b_i}$, i = 1,2,3... and $\sum \mathbf{b_i}$ converges. Prove that $\sum_{\mathbf{i=1}}^{\infty} \sum_{\mathbf{j=1}}^{\infty} \mathbf{a_{i}}_j = \sum_{\mathbf{j=1}}^{\infty} \sum_{\mathbf{i=1}}^{\infty} \mathbf{a_{i}}_j$.

8. f , for some x there are constants $\delta > \mathbf{0}\ \mathbf{a}nd\ \mathbf{M} < \infty$ such that $\left|\mathbf{f}\left(\mathbf{x}+\mathbf{t}\right) - \mathbf{f}(\mathbf{x})\right| \leq \mathbf{M}\left|\mathbf{t}\right|$ for all t $\in (-\delta, \delta)$ then prove that $\mathbf{l}im_{\mathbf{n}\to\infty} \mathbf{S_N}\left(\mathbf{f};;\mathbf{x}\right) = \mathbf{f}(\mathbf{x})$.

Contd...

Section C

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Prove that every interval is measurable.
10. State and Prove Fatou's Lemma.
11. State and Prove Stone-Weierstrass theorem.
12. If f is a positive function on 0,∞ such that
    (a) f(x+1) = x f(x) (b) f 1 (=1) (c) log f is convex ,
    then prove that f(x) = Γ (x).

Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. Prove that the class M is a σ-algebra.